Benchmarking convective dynamos
sub-grid scales modeling effects

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1. Introduction

2. Implicit vs Explicit dissipation:
modeling subgrid-scales effects

3. Analysis of energy transfers
between scales

4. Conclusions

Special thanks: A.S. Brun, S. Mathis, and the ASH and EULAG teams
THE MANY SCALES OF SOLAR MAGNETISM

Sun
Size ~ 700 Mm
Rotation ~ month
Cycle ~ 11 years

Granules
Size ~ 1 Mm
Life ~ 10 minutes

Spots
Size ~ 10 Mm
Life ~ days
**CHALLENGE: ab-initio models of convective dynamos**

Tremendously high Reynolds (flow), Taylor (rotation), and Rayleigh (buoyancy, heat) numbers.

[Diagram showing diffusion coefficients $\nu$, $\eta$, and $\kappa$ as functions of $r/R_*$ for a reference Sun. Credit: A.S. Brun]
**Challenge: ab-initio models of convective dynamos**

Tremendously high Reynolds (flow), Taylor (rotation), and Rayleigh (buoyancy, heat) numbers

Help of the so-called Large-Eddy Simulation (LES) techniques to consider much higher dissipation coefficients

![Graph showing diffusivities as a function of radial distance](cred. A.S. Brun)
VARIETY OF CONVECTIVE DYNAMOS TODAY

Variety of convective dynamos today

[Variety of convective dynamos today]

[Variety of convective dynamos today]

[Variety of convective dynamos today]

[Variety of convective dynamos today]

[Variety of convective dynamos today]

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[Variety of convective dynamos today]

[Variety of convective dynamos today]
Variety of convective dynamos today

[Kapyla+12, Warnecke+14]

[Augustson+15]

[Fan+14]

[Nelson+13]

[Brown+11]

[Racine+11]

and much more!
BENCHMARKING CONVECTIVE DYNAMO SIMULATIONS
A FIRST TAKE ON CONVECTION
A SET OF ANELASTIC MHD EQUATIONS

\[ \begin{align*}
\frac{d}{dt} u &= -\nabla \left( \frac{p}{\bar{\rho}} \right) - \frac{S}{c_p} g - 2\Omega \times u + \frac{1}{\bar{\rho}} J \times B + \frac{1}{\bar{\rho}} \nabla \cdot D_v \\
\frac{d}{dt} S &= - (\mathbf{v} \cdot \nabla) S_a - \frac{S}{\tau} + Q_{\kappa, \nu, \eta} \\
\frac{d}{dt} B &= (\mathbf{B} \cdot \nabla) u - (\nabla \cdot u) B + \nabla \cdot D_\eta \\
\nabla \cdot (\bar{\rho} u) &= 0 \\
\nabla \cdot B &= 0
\end{align*} \]

Background state based on the anelastic benchmark of Jones+11
A set of anelastic MHD equations

\[ \frac{dt}{\tau} u = -\nabla \left( \frac{p}{\rho} \right) - \frac{S}{c_p} g - 2\Omega \times u + \frac{1}{\rho} J \times B + \frac{1}{\rho} \nabla \cdot D_{\nu} \]

\[ \frac{dt}{\tau} S = - (\mathbf{v} \cdot \nabla) S_a - \frac{S}{\tau} + Q_{\kappa, \nu, \eta} \]

\[ \frac{dt}{\tau} B = (\mathbf{B} \cdot \nabla) u - (\nabla \cdot u) B + \nabla \cdot D_{\eta} \]

\[ \nabla \cdot (\bar{\rho} u) = 0 \]

\[ \nabla \cdot B = 0 \]

Background state based on the anelastic benchmark of Jones+11
A set of anelastic MHD equations

\[ \frac{d_t u}{\rho} = -\nabla \left( \frac{p}{\rho} \right) - \frac{S}{c_p} \mathbf{g} - 2\Omega \times \mathbf{u} + \frac{1}{\rho} \mathbf{J} \times \mathbf{B} + \frac{1}{\rho} \nabla \cdot \mathbf{D}_\nu \]

\[ \frac{d_t S}{\rho} = - (\mathbf{v} \cdot \nabla) S_a - \frac{S}{\tau} + Q_{k,\nu,\eta} \]

\[ \frac{d_t B}{\rho} = (\mathbf{B} \cdot \nabla) u - (\nabla \cdot u) \mathbf{B} + \nabla \cdot \mathbf{D}_\eta \]

\[ \tau \sim 20 \text{ days} \]

\[ \nabla \cdot (\bar{\rho} u) = 0 \]

\[ \nabla \cdot B = 0 \]

Dissipation

+ Sub-grid scales effects

Background state based on the anelastic benchmark of Jones+11
MODELING THE EFFECT OF UNRESOLVED SCALES
MODELING THE EFFECT OF UNRESOLVED SCALES

Resolve it with a separate adequate set of equations

*Hardest, yet to be done for convective dynamos*
Modeling the effect of unresolved scales

'Dissipative', model it as effective dissipation coefficients

Eddy viscosity
(dynamiс) Smagorinsky

Resolve it with a separate adequate set of equations

Hardest, yet to be done for convective dynamos
MODELING THE EFFECT OF UNRESOLVED SCALES

Resolve it with a separate adequate set of equations

Hardest, yet to be done for convective dynamos

'Dissipative', model it as effective dissipation coefficients

Eddy viscosity (dynamic) Smagorinsky

Minimize it as much as possible through ad-hoc numerical methods

Implicit-LES
Modeling the effect of unresolved scales

'Approach'

Resolve it with a separate adequate set of equations

Hardest, yet to be done for convective dynamos

Model it as effective dissipation coefficients

Eddy viscosity
(dynmic) Smagorinsky

Minimize it as much as possible through ad-hoc numerical methods

Implicit-LES
ASH

Enhanced diffusion
(& dynamic Smagorinsky)
(Pseudo-)Spectral Method

[Brun+ 2013]
ASH

Enhanced diffusion
(& dynamic Smagorinsky)
(Pseudo-)Spectral Method

EULAG-MHD

Implicit diffusion
(& explicit diffusion)
Finite volumes method

[Brun+ 2013]
[cred. N. Lawson]
Hydrodynamic case with enhanced diffusivities

Parameters

\( \text{Ro}_c \sim 0.06 \)
\( \text{Ta} \sim 9 \times 10^5 \)
\( \text{Ra} \sim 5.6 \times 10^4 \)
\( \text{Re} \sim 30 \)
\( \Delta \rho = 1.5 \)
\( \text{Pr} = 1 \)
\( \nu \propto \rho^{-0.5} \)

BCs
Stress-free, impenetrable

Resolution
\( N_r, N_\theta, N_\phi = 51, 64, 128 \)
**Kinetic energy balance: spectral decomposition**

\[ \partial_t E^K_L = C_{L\pm 1} + P_L + G_L + V_L + \sum_{L_1, L_2} R_{L_1,L_2} \]

- Coriolis
- Buoyancy
- Reynolds stress
- Pressure
- Viscous
- Gradient
- \& subgrid model

[cf Strugarek+13]
Kinetic energy balance: spectral decomposition

\[ \partial_t E^K_L = C_{L\pm 1} + P_L + G_L + \nu_L + \sum_{L_1, L_2} \mathcal{R}_{L_1, L_2} \]

- Coriolis
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- Viscous
- & subgrid model

Clebsch-Gordan coefficients

[cf Strugarek+13]
Kinetic energy balance: spectral decomposition

\[ \partial_t E^K_L = C_{L\pm 1} + P_L + G_L + V_L + \sum_{L_1, L_2} R_{L_1, L_2} \]

Statistical steady-state:

\[ -V_L = C_{L\pm 1} + P_L + G_L + \sum_{L_1, L_2} R_{L_1, L_2} \]

Coriolis  Buoyancy  Reynolds stress
Pressure  Viscous  & subgrid model
Gradient

Clebsch-Gordan coefficients

[cf Strugarek+13]
Kinetic energy balance & explicit dissipation
**Kinetic Energy Balance & Explicit Dissipation**

![Diagram showing kinetic energy balance at different radial positions](image)

- **KE balance @ r/R = 0.853**
- **KE balance @ r/R = 0.856**

**Legend:**
- **Coriolis**
- **Grad P**
- **Buoyancy**
- **Advec**
- **Diff**
- **Tot**

Solar Dynamo Frontiers Workshop, Boulder, 10/06/2015

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FROM LES TO ILES WITH EULAG

\[ \nu_r \quad \nu_r / 6 \quad \nu ? \]

ILES

2.3 - 2.5 \( \Omega_{\text{sun}} \)

V decreases

\[ V_r \]

\[ \pm 20 \text{ m/s} \]
FROM LES TO ILES WITH EULAG

$v_r$

$v_r/6$

2.3 - 2.5 $\Omega_{\text{sun}}$

v decreases

$V_r$

± 20 m/s

a priori dissipative $v$?
Kinetic energy balance & implicit dissipation

KE balance @ $r/R = 0.856$

LES

ILES

Coriolis
Grad P
Buoyancy
Advec
Diff
Tot

Coriolis
Grad P
Buoyancy
Advec
Diff
Tot
Estimating the implicit SGS model in EULAG

\[- \nabla_L^{SGS} = \int_S \nabla \cdot \left[ 2 \bar{\rho} \nu_{\text{eff}}(r, L) \left( \epsilon - \frac{1}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) \right]_L \cdot \mathbf{u}_L \, d\Omega \]
Estimating the implicit SGS model in EULAG

\[-\mathbf{v}_{LSGS} = \int_S \nabla \cdot \left[ 2\bar{\rho} \nu_{\text{eff}}(r, L) \left( \epsilon - \frac{1}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) \right]_L \cdot \mathbf{u}_L \, d\Omega \]
Estimating the implicit SGS model in EULAG

\[-\nu_{L}^{SGS} = \int_{S} \nabla \cdot \left[ 2\bar{\rho} \nu_{\text{eff}}(r, L) \left( \epsilon - \frac{1}{3} (\nabla \cdot \mathbf{u}) I \right) \right] \cdot u_{L} \, d\Omega\]
Estimating the implicit SGS model in EULAG

\[-\nu_{SGS}^L = \int_S \nabla \cdot \left[ \frac{2\tilde{\rho}}{\nu_{eff}(r, L)} \left( \epsilon - \frac{1}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) \right] \cdot \mathbf{u}_L \, d\Omega\]

Use this parametrized $\nu_{eff}$ in an ASH simulation.
**Reconciling implicit & explicit SGS models**

Possible origins of the remaining differences:
- Effective Prandtl number \( \neq 1 \) in the ILES simulation?
- The ILES is not *a priori* equivalent to viscous stresses…

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**Solar Dynamo Frontiers Workshop, Boulder, 10/06/2015**

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Prandtl number effect in LES–ILES comparisons

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**Kinetic Energy spectrum**

- **ASH**
- **ASH Pr=2**
- **ILES**

<table>
<thead>
<tr>
<th>r/R</th>
<th>KE (r/R = 0.952)</th>
<th>KE (r/R = 0.904)</th>
<th>KE (r/R = 0.856)</th>
<th>KE (r/R = 0.808)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^7</td>
<td>10^6</td>
<td>10^5</td>
<td>10^4</td>
<td>10^3</td>
</tr>
</tbody>
</table>

**Entropy spectrum**

- **ASH**
- **ASH Pr=2**
- **ILES**

<table>
<thead>
<tr>
<th>r/R</th>
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Prandtl number effect in LES–ILES comparisons

**Kinetic Energy spectrum**

ASH
ASH Pr=2
ILES

**Entropy spectrum**

ASH
ASH Pr=2
ILES

r/R = 0.856

-5/3
PRANDTL NUMBER EFFECT IN LES–ILES COMPARISONS

Kinetic Energy spectrum

\[
\frac{r}{R} = 0.856
\]

\[ KE \]

\[-5/3 \]

Entropy spectrum

\[
\frac{r}{R} = 0.856
\]

\[ S \]

\[-5/3 \]
Dynamo: a preliminary test case with ASH

Parameters

\[ \begin{align*}
\text{Ro}_c & \sim 0.25 \\
\text{Ta} & \sim 6.3 \times 10^6 \\
\text{Ra} & \sim 1.6 \times 10^6 \\
\text{Re} & \sim 200 \\
\Delta \Omega & \sim 25 \\
\text{Pr} & = 1 \\
\text{Pm} & = 1 \\
\nu & \propto \Omega^{-0.5}
\end{align*} \]

ongoing work to improve compatibility of magnetic BCs between ASH and EULAG
**CONCLUSIONS & PERSPECTIVES**

A *convective dynamo benchmark* has been successfully developed to specifically study the impact of sub-grid scales modeling (dynamo comparison ongoing)

A *promising method* based on *spectral transfers analysis* is proposed to relate *implicit* and *explicit* sub-grid scales models

Refinements of the spectral analysis method could be developed to push the comparison further (*radial dependency, full* $\nu_{\text{eff}}[r,l,m]$)

The method can easily be further generalized to derive *thermal/radiative* and *ohmic dissipation coefficients*

Comparisons with other sub-grid scale modeling methods, such as *dynamic Smagorinsky method*, will be explored in a near future