THE MAGNETIC SHEAR-CURRENT EFFECT: APPLICATIONS TO SPACE AND ASTROPHYSICAL PLASMAS

J. SQUIRE and A. BHATTACHARJEE

PRINCETON PLASMA PHYSICS LABORATORY, PRINCETON UNIVERSITY
MEAN-FIELD DYNAMO

- Interested in the *large-scale dynamo*

- Mean-field average — extracts the “large scales” from turbulence.

\[
\frac{\partial B_T}{\partial t} = \nabla \times (U_T \times B_T) + \eta \nabla^2 B_T
\]

\[
\varnothing_B = B + b
\]

\[
\varnothing_{\bar{U}} = \bar{U}
\]

\[
\frac{\partial B}{\partial t} = \nabla \times (U \times B) + \nabla \times \mathcal{E} + \eta \nabla^2 B
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U_T = U + u
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\[
\frac{\partial B}{\partial t} = \nabla \times (U \times B) + \nabla \times \mathcal{E} + \eta \nabla^2 B
\]

\[
\mathcal{E}(B, U) = u \times b
\]
If the $u$ and $b$ are “small scale” compared to $B$ and $U$

AND

$B$ and $U$ are a small perturbation to the turbulence

$$\mathcal{E}(B, U) = \alpha B + \beta \nabla B + \gamma \nabla U + \cdots$$

If $B=0$ implies $b=0$, this is a kinematic dynamo $\alpha = \alpha(u, B, U)$

The “$\alpha$ effect” can cause a dynamo,

But the $\alpha$ effect is only possible if something breaks symmetry; e.g., helicity $\int dx \ u \cdot \nabla \times u \neq 0$.

The “$\beta$ effect” usually acts as a turbulent resistivity; i.e., a damping of the large-scale field.

More interesting $\beta$ effects are possible.
Challenges to Kinematic Mean-Field Dynamo Theory

- At even moderate $Rm$, the fast-growing small-scale dynamo implies that velocity fluctuations should always be accompanied by magnetic field fluctuations of a similar magnitude (*Schekochihin et al.* 2004), questioning the relevance of the classical kinematic theory.
The simplest relevant system exhibiting MRI turbulence is the local incompressible MHD equations — remove global curvature.

In the *shearing box*, boundary conditions are periodic in $y$ (azimuthal) and $z$ (vertical), and shearing periodic in $x$ (radial).
- We use the horizontal average for the mean-field average.

- Study the dynamo by studying \( \mathcal{E}(B, U) \).

- Mean fields depend on \( z \):
  \[
  B(z, t)
  \]

- Include a mean shear flow
  Shearing box BCs

\[
U_0 = -S x \hat{y}
\]
New dynamo mechanism — *the magnetic shear-current effect* — small-scale magnetic fields have a *positive* effect on the large-scale dynamo.

Effect requires velocity shear (e.g., Keplerian).

No $\alpha$ effect required.

Off-diagonal component of $\beta$ couples with the shear.
WHY SHOULD WE CARE?

1. Accretion disk dynamo — magnetic shear current effect seems a likely candidate for central (unstratified) dynamo.

Background
nonhelical shear dynamos

Low Rm
no small-scale dynamo

High Rm
with small-scale dynamo

Analytic closure

The magnetic SC effect as the MRI dynamo
Homogenous fluctuations do not contribute to $\nabla \times \mathcal{E}$

$$\nabla \times \mathcal{E} = \left( \partial_z \langle u_x b_z - b_x u_z \rangle, \partial_z \langle u_y b_z - b_y u_z \rangle, 0 \right)$$

From

$$\partial_t u \sim -(u \cdot \nabla U + U \cdot \nabla u) + (b \cdot \nabla B + B \cdot \nabla b)$$

$$\partial_t b \sim \nabla \times (u \times B) + \nabla \times (U \times b)$$

$$\nabla \times \mathcal{E} = \nabla \times (u_h \times b_i + u_i \times b_h)$$

Inhomogeneous $b$ from $\nabla \times (u_h \times B)$

This is the kinematic dynamo

Inhomogeneous $u$ from $(b_h \cdot \nabla B + B \cdot \nabla b_h)$

Dropped terms like $\nabla \times (u \times b - u \times b)$
NONHELICAL SHEAR DYNAMOS

- We can run a simulation and (maybe) see a dynamo — but how do we know the cause?

Yousef et al. 2008

Brandenburg et al. 2008

Singh et al. 2015
NONHELICAL SHEAR DYNAMOS

- Mechanism for nonhelical shear dynamos has been controversial.

**Shear-current effect**

Off diagonal $\beta$ term provides “negative resistivity”.

Rogachevskii & Kleeorin 2003; Urpin 1999

**Stochastic-$\alpha$ effect**

Fluctuations in $\alpha$ (with $\langle \alpha(t) \rangle = 0$) cause $\langle B^2 \rangle$ to grow exponentially, even though $\langle B \rangle = 0$.

Vishniac & Brandenburg 1996; Heinemann et al. 2011

- Role of helicity flux also?

Vishniac & Cho 2001; Shapovalov & Vishniac 2011; Ebrahimi & Bhattacharjee 2015
Mean-field evolution $\mathbf{B}(z, t)$

$$\partial_t B_x = -\alpha_{yx} \partial_z B_x - \alpha_{yy} \partial_z B_y - \eta_{yx} \partial_z^2 B_y + (\eta_{yy} + \bar{\eta}) \partial_z^2 B_x \tag{1}$$

$$\partial_t B_y = -S B_x + \alpha_{xx} \partial_z B_x + \alpha_{xy} \partial_z B_y - \eta_{xy} \partial_z^2 B_x + (\eta_{xx} + \bar{\eta}) \partial_z^2 B_y, \tag{2}$$

\(\eta\) is \(\beta\)

Shear-current - coherent growth $\mathbf{B}(z, t) = \mathbf{B} e^{ikz} e^{\gamma t}$

$$\gamma_\eta = k \sqrt{\eta_{yx} (-S + k^2 \eta_{xy})} - k^2 \eta_t \tag{3}$$

Sign of \(\eta\) is key!

Stochastic alpha - incoherent growth $\langle \mathbf{B} \rangle = 0$, $\langle \mathbf{B}^2 \rangle$ grows

$$\gamma_\alpha = \left( \frac{k^2 S^2 D_{yy}}{2} \right)^{1/3} - k^2 \eta_t \tag{4}$$

$$\langle \alpha_{yy}(t) \alpha_{yy}(t') \rangle = D_{yy} \delta(t - t')$$

Not too important
KINEMATIC DYNAMO?


all find that the kinematic shear-current effect has the wrong sign, contrary to Rogachevskii & Kleeorin

Where does the dynamo come from?

Stochastic-$\alpha$ effect?

SOMETIMES
KINEMATIC DYNAMO?  Low Rm

- Important to understand the kinematic dynamo before looking at magnetic case.

- No small-scale dynamo.

- Stochastic-$\alpha$ effect is responsible for Yousef dynamo

**BUT**

*Rotation is important*
LOW RM

VELOCITY FORCING (KINEMATIC)

NONLINEAR

QUASI-LINEAR

CE2

$L_z \gg L_x, L_y$

$E_B$

$10^{-8}$ $10^{-6}$ $10^{-4}$ $10^{-2}$ $10^{0}$ $10^{2}$ $10^{4}$ $10^{6}$

$t$

$10^{0}$ $10^{10}$ $10^{20}$ $10^{30}$ $10^{40}$

$E_B$

$t$

Rm=100
KINEMATIC DYNAMO?

- Without rotation, kinematic shear-current effect has the wrong sign.

  Still get dynamo due to stochastic-$\alpha$
  This depends on horizontal size!

- Anticyclonic (e.g., Keplerian) rotation changes this — still very slow growth.

- Analytic results agree — this is Rädler effect,

\[
(\eta_{yx})_u = (S - 2\Omega) \Xi \\
^{\text{Keplerian}} \Omega = \frac{2}{3} S
\]

- Trends seen in Yousef et al. (2008).
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Keplerian \( \Omega = \frac{2}{3} S \)

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- Still get dynamo due to stochastic-\(\alpha\) (\(\eta_{xy} = \frac{2}{3} S\)).

\[ \eta_{xx}/\eta_0, \eta_{yx}/\eta_0 \]

\[ < 0 \]
LET'S ADD INDUCTION FORCING

Generates $b_h$

CE2

No small-scale dynamo, since quasi-linear
LET'S ADD INDUCTION FORCING

Nonlinear DNS  \( R_{m} = 100 \)

- a) \( f_{b} = 0 \)
- b) \( f_{b} = 0.1f \)
- c) \( f_{b} = 0.2f \)
- d) \( f_{b} = 0.5f \)
SMALL-SCALE MAGNETIC FIELDS DRIVE THE DYNAMO

- As $b_h$ increases compared to $u_h$, dynamo growth rate increases.

This is due to a change in sign $\eta_{yx}$.

$$(\eta_{yx})_u > 0 \quad + \quad (\eta_{yx})_b < 0 \quad \rightarrow \quad (\eta_{yx})_{total} < 0$$
THIS AGREES WITH ANALYTIC RESULT

\[
\left( \eta_{yx} \right)^S_u = \int d\omega \, dk \, \frac{32\pi k^2 W_u(k, \omega) \omega^2 \tilde{\eta}^2}{15 \left( \tilde{\eta}^2 + \omega^2 \right)^2 \left( \tilde{\nu}^2 + \omega^2 \right)} \] > 0

\[
\left( \eta_{yx} \right)^S_b = \int d\omega \, dk \, 8\pi k^2 \rho^{-1} W_b (k, \omega) \frac{4\omega^4}{15 \left( \tilde{\nu}^2 + \omega^2 \right)^3} \]
\[ - \frac{2\tilde{\eta}\tilde{\nu}^3 + \tilde{\eta}^2\tilde{\nu}^2 + 2\omega^2\tilde{\eta}^2 + 3\omega^4}{15 \left( \tilde{\eta}^2 + \omega^2 \right) \left( \tilde{\nu}^2 + \omega^2 \right)^2} \] < 0
\[
\left| \left( \eta_{yx} \right)^S_u \right| \ll \left| \left( \eta_{yx} \right)^S_b \right| \]
\[ + \frac{4\omega^2 \tilde{\eta}\tilde{\nu}}{15 \left( \tilde{\eta}^2 + \omega^2 \right)^2 \left( \tilde{\nu}^2 + \omega^2 \right)} \] \[ \tilde{\eta} = k^2 \tilde{\eta} \]

- And agrees with the $\tau$ approximation (Rogachevskii & Kleeorin 2004)

- And agrees with perturbative MRI calculations (Lesur & Ogilvie 2008)

- Shows that the MSC effect is caused by the fluid pressure response
INTERESTING — BUT

FORCING THE INDUCTION EQUATION?

Can the small-scale dynamo drive the large-scale dynamo?
Small scale dynamo is always present at moderate Rm

Magnetorotational turbulence – instability develops with $b \sim u$. 

From Schekochihin et al 2007
Fast growth of small-scale dynamo, saturates $t \approx 40$.

Large-scale dynamo driven by small-scale $b$ fluctuations?

Large-scale dynamo saturates — change in $\eta$?
High Rm

NO ROTATION

KEPLERIAN ROTATION

After small-scale saturation

Kinematic
ACCRETION DISK DYNAMO

Is the magnetic shear-current effect related to the accretion disk dynamo?

- MRI turbulence has $b_{rms} > u_{rms}$.

- The Keplerian rotation should be favorable to the dynamo.

- Driven simulations show cycles — similar to MRI dynamo cycles?
Some of our best evidence comes from CE2, driven with $u$ and $b$, in the saturated regime.

Squire & Bhattacharjee PRL (2015)
CE2 IN THE SATURATED REGIME

Saturated dynamo depends strongly on $P_m$
So does self-sustaining MRI turbulence

Constant Rm

This dependence is opposite to the linear system: *Increase in dissipation causes increase in turbulence level.*
1. The coherent large-scale dynamo is (at least partially) responsible for the Pm dependence of MRI turbulence:

   **CE2 shows a similar Pm dependence to nonlinear MRI turbulence.**

   **The only possible reason for this is the coherent dynamo.**

2. The dynamo mechanism is the magnetic shear-current effect:

   **The kinematic effect is too weak to drive the observed dynamo.**
CONCLUSIONS

- Magnetic shear-current effect is like inverse quenching — small-scale dynamo can drive a large-scale dynamo.

  *Agreement between simulation and analytic results.*

- Good evidence that magnetic shear-current effect is responsible for unstratified MRI dynamo (Shi, Stone, and Huang 2016)

  *Shear flows being ubiquitous, is the magnetic shear-current effect important for the Sun?* (Hotta, Rempel, and Yokoyama, 2016)