Rotating Convection
Robert Ecke, Center for Nonlinear Studies
Los Alamos National Laboratory

Sakai 1997

R. Stevens
Solar Convection
Ek ~ $10^{-12}$
Ra ~ $10^{20}$
Pr ~ $10^{-6}$
Ra/Ra_c ~ $10^3$

Jupiter
Ek ~ $10^{-12}$
Ra ~ $10^{24}$
Pr ~ 1
Ra/Ra_c ~ $10^7$

Earth's Outer Core
Ek ~ $10^{-15}$
Ra ~ $10^{25}$
Pr ~ 0.01-1
Ra/Ra_c ~ $10^4$
What is convection and how is heat transported in a fluid?

Turbulence and boundary layers

How does rotation influence convection?

Geostrophic turbulence as a new frontier: Challenges and opportunities.
Rayleigh-Bénard Convection

Rayleigh Number

\[ Ra = \frac{g \alpha \Delta T d^3}{\nu \kappa} > Ra_c = 1708 \]

Prandtl Number

\[ Pr = \frac{\nu}{\kappa} \]

- Sun: $10^{-6}$
- Mercury: 0.015
- Ideal Gas: 0.7
- Water: 6
- Motor Oil: $> 10^{-10}$
- Mantle: $10^{25}$
With Rotation

\[ \Omega^* = \Omega d^2 / \nu \]

Ekman Number

\[ Ek = 1 / (2\Omega^*) \]

Taylor Number

\[ Ta = (2\Omega^*)^2 \]

\[ Ek = 1 / \sqrt{Ta} \]
Suppression of Convective Onset

Convection

\[ \text{Ra}_c \sim \text{Ta}^{2/3} \]

Conduction

\[ \lambda/d \sim \text{Ta}^{-1/6} \]

Chandrasekhar 1953
Linear Stability Analysis

Convective Rossby Number

buoyancy versus rotation

\[ \tau_{\text{buoy}} \sim \sqrt{d/(g\alpha\Delta T)} \]

\[ Ro = \frac{\tau_{\text{rot}}}{\tau_{\text{buoy}}} \]

\[ Ro = \sqrt{\frac{Ra}{PrTa}} \]

Rotation wins: \( Ro \ll 1 \)

Buoyancy wins: \( Ro \gg 1 \)
Structure of flow near transition to turbulence

Sakai JFM 1997

\[ \Gamma = \frac{L}{H} = 3.3 \quad \text{Square} \]

\[ Ra \approx 8 \times 10^6 \]

\[ Ta \approx 1 \times 10^8 \]

\[ Ro \approx 0.12 \]
A 18.73 cm diameter container of water (Pr$_7$). The convection in these cases was driven by a fixed 10 W heating power, corresponding to a flux Rayleigh number $ Ra_F = Ra = \frac{E^2}{Pr} \sim 10^{12}$. Even at this relatively low heating power, it is not possible, with water as the working fluid, to reach low enough $Ra = Ra_{crit}$ needed to access the cellular regime in the RoMag device. This inability to reach low $Ra = Ra_{crit}$ in low $E$ cases in Pr$_7$ fluids is a problem for all current laboratory rotating convection devices (cf. Ecke and Niemela, 2014; Cheng et al., 2015). (The opposite issue arises in Pr$_{10}$ liquid metals, where it is more difficult to access $Ra = Ra_{crit}$ (e.g., Cioni et al., 2000; King and Aurnou, 2013; Ribeiro et al., 2015)).

The three laboratory images in the top row correspond to rotation rates of 60 rpm ($E = 1.2 \times 10^7$), 10 rpm ($E = 7.5 \times 10^6$) and 4 rpm ($E = 1.9 \times 10^6$), from left to right. Since the rotation rate, and therefore $E$, is varied in each laboratory case, this leads to normalized domain widths of $W'_c$ for the leftmost case; $W'_c$ for the middle case; and $W'_c$ for the rightmost case.

The second row in Fig. 7 shows the temperature fluctuation field, $h$, from non-slip DNS cases (Stellmach et al., 2014). The left and right images in this row have Pr$_1$, while the middle two images have Pr$_7$. The color scale is such that vermilion structures are warmer than the surrounding fluid and aquamarine structures are cooler than the surrounding fluid. This color scale is used for $h$ throughout this manuscript. The horizontal scale of the numerical domain is $W'_c$ for all DNS cases reported.

Fig. 8 shows images from cases with free-slip MBCs. The top row of images from DNS made with nearly the same parameters as the non-slip DNS cases shown in Fig. 7. The bottom row shows images from the reduced models (e.g., Sprague et al., 2006; Rubio et al., 2014). Note that Pr$_7$ in the left three images in this row, whereas $Ra = Ra_{crit} = 16.1$ in the right two cases, but the Prandtl number is lowered to Pr$_1$ in order to boost the effective buoyancy forcing in the rightmost case shown. The horizontal scale of the numerical domain is $W'_c$ for all of the reduced modeling results reported.

Fig. 7: Snapshot visualizations showing the cellular, Taylor column, plume and geostrophic turbulence regimes, as defined in Sprague et al. (2006) and Julien et al. (2012b). Top row: flake visualizations from the laboratory experiments made in Cheng et al. (2015). Bottom row: temperature fluctuation field $h$ from the non-slip DNS cases in Stellmach et al. (2014).
Heat transport, Structure, and Parameter Space
Heat Transport

$\Omega = 0$

 enhancement

 suppression

Zhong, Ecke, Steinberg: JFM 1993
Turbulent heat transport continued

In a typical apparatus. The ratio $\frac{\text{Nu}(\Omega)}{\text{Nu}(0)}$ is shown as a function of the rotation rate $\Omega$. Red open squares: $10^{-15/4}$, Green solid $10^{-13/4}$, Purple solid squares: $10^{-1/3}$, Black stars: $10^{-1/2}$, Blue open squares: $10^{-1/3}$, Black open circles: $10^{-1/4}$.

In Fig. 3 (color online), the data must be averaged over an additional three or four hours and used to eliminate heat flux fluctuations. Those results closely agree with previous results [1] (experiment). Black open circles: $10^{-1/4}$, Red solid circles: $10^{-1/3}$, Blue solid circles: $10^{-1/2}$, Green open diamonds: $10^{-1/4}$.

The enhancement of $\text{Nu}$ is shown in the presence of rotation to occur at large $\text{Ro}$ (small $\text{Pr}$). At large $\text{Ro}$ (small $\text{Pr}$), the maximum of $\text{Nu}$ due to modest rotation is clearly seen at all $\text{Pr}$. In Fig. 2 (color online), the data must be averaged over an additional three or four hours, and temperatures and heat currents were then averaged over an additional three or four hours, and experimental uncertainties are typically no larger than the size of the symbols.

The turbulent heat transport continued as expected. The maximum of $\text{Nu}$ due to modest rotation is clearly seen at all $\text{Pr}$. The enhancement of $\text{Nu}$ is shown in the presence of rotation to occur at large $\text{Ro}$ (small $\text{Pr}$). At large $\text{Ro}$ (small $\text{Pr}$), the maximum of $\text{Nu}$ due to modest rotation is clearly seen at all $\text{Pr}$.

Zhong et al., PRL 2009
Convection suppressed by rotation but heat transport enhanced by rotation!!???

What’s going on?
Large scale circulation

Vortices dominant

Vorobieff & Ecke JFM 2002
No Rotation

Thermal plumes driven by buoyancy

Hot fluid accumulates mass and momentum forming thermal plume

Strong Rotation: Ekman suction

Hot fluid actively pumped out of the boundary layer
Phase Diagram \( Pr \approx 5 \)

- Buoyancy dominated
- Rotation dominated
- Rotation influenced
- Weak convection
- Conduction

Experiments
- Rossby 1969
- Pfotenhauer et al, 1987
- Zhong et al, 1993
- King et al, 2009
- Zhong et al, 2009
- Zhong-Ahlers, 2010
- Niemela, Babuin et al, 2010
- Kunnen et al, 2013

Numerical Simulation
- Julien et al, 1996
- Stevens et al, 2009
- Grooms et al, 2010
- Julien et al, 2012
An interesting region ...
The transition from rotationally controlled to non-rotating heat transfer behaviour is shown in Fig. 2. The Nusselt number is defined as $\frac{Nu}{Nu_{\text{c}}} \propto \left( \frac{Ra}{Ra_{\text{c}}} \right)^{\frac{2}{7}}$.

The dynamical transition at $\frac{\Omega}{T_0} \times 10^5$ (Fig. 3b). Following refs 17, 27, we define the transitional Nusselt number as $Nu_{\text{c}}$. Several different fluids are used, as characterized by the Prandtl number, Pr. 

In this convective boundary layer, the thermal boundary layer is thinner than the Ekman layer, and therefore mixing truncates the influence of the Ekman layer, and therefore the uppermost part of the Ekman layer is mixed with the bulk. This region is the Ekman boundary layer.

The numerically determined non-dimensional Ekman layer thickness, $\frac{\delta_E}{\lambda_E}$, scaling by equating $Nu_{\text{c}}$ and $\delta_{\text{c}}$, where $\delta_{\text{c}}$ stabilizing effect of the Coriolis force in the laboratory experiments yields $Nu_{\text{c}}$.

Non-rotating Rayleigh number: $Ra_{\text{c}} = 10^4$, Ekman number: $Ek_{\text{c}} = 10^{-5}$, Ekman boundary layer by varying the rotation rate. To test our hypothesis, we solve for a transitional Rayleigh number, $Ra_{\text{c}}$, where $Ra_{\text{c}}$ - 3 decades in $Ra$. Solid black lines represent the non-rotating scaling law $Nu_{\text{c}} \propto Ra_{\text{c}}^{2/7}$, with no-slip top and bottom boundaries and periodic sidewalls. Gravity is applied using water (Pr = 100).

More specifically, heat transfer behaviour is shown in Fig. 2.

$Nu_{\text{c}} \propto \left( \frac{Ra}{Ra_{\text{c}}} \right)^{\frac{2}{7}}$.
Asymptotic equations:

\[ \Lambda a \rightarrow \infty \quad E_k \rightarrow 0 \]

Taylor Columns

\[ \text{Nu} - 1 = C_{Pr}^{1/2} E_k^2 R_a^{3/2} \]

Geostrophic Turbulence

\[ \text{Ra}/\text{Ra}_c = 18.4 \]

Grooms et al, PRL 2010

Julien et al, Geo. Astro Fluid Dyn. 2011

Julien et al, PRL 2012
Low Prandtl Number Results (Ecke/Niemela)

- Gaseous helium @ 4.5 K
- $\text{Nu}_0 = 0.120 \, \text{Ra}^{0.31}$
- $\text{Pr} = 0.7$
- $\Gamma = \text{D/H} = 1/2$
- $\text{Ra}$ range: $4 \times 10^9 - 4 \times 10^{11}$
- $\text{Ek}$ range: $2 \times 10^{-7} - 3 \times 10^{-5}$
- $\text{Nu}$ range: 30 - 450
Normalized Heat Transport

Ra = 6.2 x 10^9

Increasing Ω
Data Collapse and Scalings

- **Ro collapse fair**: $Ro_H = 0.35$, $Ro_L = 0.12$
- **RaEk$^{7/4}$ collapse better**: $RaEk^{7/4}_H = 2$, $RaEk^{7/4}_L = 0.5$

Ecke & Niemela PRL 2014
 Ek
Ro / Ra = 2, Pr = 6

Buoyancy Dominated

Ro = 0.35, Pr = 0.7

Rotation Influenced BLs

Buoyancy Influenced

Geostrophic

Ra / Ra_c = 4

Weak Convection

Ra_t = 0.5 Ek ^{-7/4}

Ek
Geraschenko, Backhaus, Ecke

Water: Pr = 5.4

No rotation

Ek = 1.0 \times 10^{-5}

Ek = 1.5 \times 10^{-5}

\lambda

Hankel

\Omega

Correlation

Ro=0.0487,Ra=6.75\times10^7,Ta=4.6\times10^9

Ro=0.0478,Ra=1.42\times10^8,Ta=1\times10^{10}

Ro=\infty,Ra=3.03\times10^7,Ta=0

\{u,w\}
$\text{Nu} = \begin{cases} 
1 & \text{Local } Ta = 2.8 \times 10^9 \\
7 & \text{Local } Ta = 4.6 \times 10^9 \\
10 & \text{Local } Ta = 1 \times 10^{10} \\
100 & \text{Global } Ta = 4.6 \times 10^9 
\end{cases}$
The excellent agreement found between DNS and the asymptotic predictions, which becomes poorer for stress-free boundary conditions and moderate Prandtl numbers. Data from laboratory experiments, denoted by stars in Fig. 1, are in line with these numerical findings. The up- and downwellings form intense columnar structures, which are less confined between no-slip boundaries. In fact, even for the stress-free case, with the regime called geostrophic turbulence (GT). Similar regimes are also observed in the stress-free case, with the regime called geostrophic turbulence (GT). Similar regimes are also observed in the stress-free case, with the regime called geostrophic turbulence (GT). Similar regimes are also observed in the stress-free case, with the regime called geostrophic turbulence (GT). Similar regimes are also observed in the stress-free case, with the regime called geostrophic turbulence (GT).
Challenges

$$Ek \sim \frac{1}{H^2} \quad Ra \sim H^3 \quad Ra_c \sim H^{8/3} \sim \Delta T_c$$

Easy to get $Ra$ close to $Ra_c$

Hard to measure global $Nu$ accurately since $Nu \sim O(1)$; parasitic heat leaks hard to account for.

Opportunities

Local heat transport possible alternative

Numerous large $Ra$ & $Ta$ experiments: Göttingen, Eindhoven, Trieste

Can we properly characterize heat transport scaling for geostrophic regime?
Fluids carry heat effectively

Without rotation, \( \text{Nu} \sim \text{Ra}^{1/3} \ (\text{Ra}^{0.31}) \), it’s the BL.

Rotation suppresses convection but can enhance heat transport - again, it’s the BL.

Geostrophic turbulence as a new frontier where it’s all about vortices ...
Extra Slides ....
Malkus Boundary Layer Theory for Rotating Convection

Well Mixed

\[ \frac{\partial}{\partial z} \approx 1 \]

\[ \frac{\partial}{\partial z} \ll 1 \]

\[ \frac{\Delta T}{2} \]

\[ \frac{\Delta T}{2} \]

Free

Rigid

\[ \frac{\delta}{d} \]

\[ \frac{z}{d} \]
\[ Ra_\delta = \frac{g \alpha \delta^3 \Delta T/2}{\nu \kappa} = \frac{Ra}{2} \left( \frac{\delta}{\delta} \right)^3 = Ra_c(Ek_\delta) = BEk_\delta^{-4/3} \]

\[ \frac{\delta}{\delta} \sim Ra^{-3} Ek^{-4} \]

\[ Nu \sim Ra^3 Ek^4 \]

\[ \frac{Ra}{Ra_c} < Ek^{-1/6} \]
Rayleigh-Bénard convection: Onset and rolls

\[ \lambda \sim 2d \]

\[ \Delta T > \Delta T_c \]
Heat transport - solid

Heat diffuses in a solid

\[
\dot{Q} = \frac{k \Delta T A}{L}
\]

\[
\frac{\partial T (\mathbf{r}, t)}{\partial t} = \kappa \nabla^2 T (\mathbf{r}, t)
\]
\[ Ra_\delta = \frac{g \alpha \delta^3 \Delta T / 2}{\nu \kappa} = \frac{Ra}{2} \left( \frac{\delta}{d} \right)^3 = Ra_c \approx 600 \]

\[ \delta / d = (2Ra_c / Ra)^{1/3} \]

\[ Nu = \frac{d}{2\delta} \sim Ra^{1/3} \]
Turbulent Heat Transport

\[ \text{Nu} \sim \text{Ra}^{0.29} \]

Zhong et al., 2009

(Liu & Ecke: PRL-1997, PRE-2009)
The beginnings ....

The instability of a layer of fluid heated below and subject to Coriolis forces

By S. Chandrasekhar, F.R.S., Yerkes Observatory

(Received 29 December 1952)


Tellus

A QUARTERLY JOURNAL OF GEOPHYSICS

A Theoretical and Experimental Study of Cellular Convection in Rotating Fluids

YOSHINARI NAKAGAWA and PAUL FRENZEN, Hydrodynamics Laboratory, University of Chicago

(Manuscript received, September 26, 1954)

Nakagawa & Frenzen, Tellus (1955)
Fig. 12. Experimental results versus theoretical curve for ordinary cellular convection in water.

Ra\(_c \sim \) Ta\(^{2/3}\)

\[
R_c = \frac{\alpha g D^4}{\kappa V}
\]

\[
T = \frac{4 \Omega^2 D^4}{\kappa V}
\]

Fig. 4. Side view of ink rising from the bottom of a cylinder of water in the ascending cores of cells driven by heating from below. Water 23.5° C; air 24.0° C; depth 18 cm; rotating 10.9 rpm; Taylor number 5.5 \times 10^4; heating plate input 5 watts. (Approx. full size.)
A study of Bénard convection with and without rotation

By H. T. Rossby†
Department of Meteorology
Massachusetts Institute of Technology

(Received 16 May 1968 and in revised form 2 October 1968)

Rigid Top & Bottom Boundaries

Heat Transport
Open Surface

Figure 16. Vortex interactions in the convective grid: 1. interlacing of two vortices; 2. formation of a double helix; 3. coalescence of the vortices; $Ra = 1.05 \times 10^9$, $Ta = 5.9 \times 10^4$.

Boubnov, Golitsyn 1986, 1990
Temperature Fluctuations

**No Rotation**

*Ro* = ∞

**Rotation**

*Ro* = 0.37

**Julien et al**

*Ro* = 0.75

\[ Pr = 1 \]

\[ Ra = 1 \times 10^8 \]

\[ Pr = 5 \]

\[ Ra = 4 \times 10^8 \]

Liu & Ecke, 2010
Velocity and Vorticity

Vorobieff & Ecke 02

Visualization & Velocity (PIV)
Temp (TCLC)

Top View

Side View

Ra = 3.2 \times 10^8
Turbulent rotating convection

- \( \Omega = 0, \ Ro \rightarrow \infty \)
  - \( \frac{R}{R_c} = 1.2 \times 10^5 \)

- \( \Omega = 9.6 \times 10^2, \ Ro = 3.89 \)
  - \( \frac{R}{R_c} = 1.5 \times 10^3 \)

- \( \Omega = 4.8 \times 10^3, \ Ro = 0.77 \)
  - \( \frac{R}{R_c} = 169 \)

- \( \Omega = 1.9 \times 10^4, \ Ro = 0.19 \)
  - \( \frac{R}{R_c} = 26 \)

- \( \Omega = 3.4 \times 10^4, \ Ro = 0.11 \)
  - \( \frac{R}{R_c} = 12 \)

- \( \Omega = 4.8 \times 10^4, \ Ro = 0.08 \)
  - \( \frac{R}{R_c} = 8 \)

Figure 4. Instantaneous temperature maps in the plane adjacent to the top of the cell for \( \Omega = 3.2 \times 10^8 \).

Dimensionless rotation rates, Rossby numbers and \( \frac{R}{R_c} \) values are labelled in the figure. Bright areas indicate highest temperature, dark areas lowest.

The flow structure near the top of the cell can be loosely categorized as follows. First, for \( \Omega = 0, \ Ro \rightarrow \infty \), the flow is dominated by sheets of thermal plumes. The plumes originating near the top surface are manifested as limit lines in instantaneous streamline patterns (e.g. one in the velocity map for \( \frac{R}{R_c} = 1.5 \times 10^3 \), figure 3). Temperature maps show these thermal plumes as lines of colder (dark) material (figure 4, top row).

The limit-line behaviour of the streamline patterns is dictated by the three-dimensional structure of the flow. Cold material is ejected from the top thermal boundary layer.
Heat Transport and Visualization - 1993

Water: Pr = 6.4

Shadowgraph

Figuhe 1. Illustration of the Rayleigh–Bénard convection cell.

Heat Transport

Zhong, Ecke, Steinberg: JFM 1993
Increasing skewness of vorticity with decreasing Ro

No Rotation

\( Ro = \infty \)

\( Ro = 0.19 \)

\( Ro = 0.75 \)

\[ Pr = 5.8 \quad Ra = 3 \times 10^8 \]

\[ Pr = 1 \quad Ra = 1 \times 10^8 \]
Transient dynamics visualized using optical shadowgraph

Zhong, Ecke, Steinberg 1993
Turbulent rotating convection

\[ \Omega = 0, \, Ro \to \infty, \quad R/R_c = 1.2 \times 10^5 \]
\[ \Omega = 9.6 \times 10^2, \, Ro = 3.89, \quad R/R_c = 1.5 \times 10^7 \]
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General form of the flow