Vortices in Multi-Dimensional Protoplanetary Disks

Hui Li (李暉  LANL)
Collaborators:
Rossby wave instability ('99-00): Lovelace, Finn, Colgate, …
Protoplanetary disk ('00-present): S. Li, W. Fu, S. Jin, R. Miranda, A. Isella, S. Lubow

Casassus et al., Nature, 2013
van der Marel et al., Science, 2013
A Bit of History ...

Stirling: Can we build “paddle wheels” in disks to transport angular momentum?

\[ v_\varphi^2 = \frac{GM}{r} + \frac{r}{\Sigma} \frac{\partial p}{\partial r} \]

Keplerian velocity

Correction for pressure gradient
“Paddle Wheels” in Disks

ROSSBY WAVE INSTABILITY OF KEPLERIAN ACCRETION DISKS

R. V. E. Lovelace
Department of Astronomy, Cornell University, Ithaca, NY 14853; rv11@cornell.edu
H. Li and S. A. Colgate
T-6, Los Alamos National Laboratory, Los Alamos, NM 87545; hli, colgate@lanl.gov
AND
A. F. Nelson
Department of Physics, The University of Arizona, Tucson, AZ 85721; andy@as.arizona.edu
Received 1998 March 19; accepted 1998 October 15

ROSSBY WAVE INSTABILITY OF THIN ACCRETION DISKS. II. DETAILED NUMERICAL SIMULATIONS

H. Li,1 J. M. Finn,2 R. V. E. Lovelace,1,3 AND S. A. Colgate
Received 1999 July 20; accepted 1999 December 3

ROSSBY WAVE INSTABILITY OF THIN ACCRETION DISKS. III. NONLINEAR EVOLUTION

H. Li,1,2 S. A. Colgate,1 B. Wendroff,3 AND R. Liska4
Received 2000 October 18; accepted 2000 December 19
**Accretion Disk Basics**

Rayleigh Criterion:

\[
\kappa^2 \equiv \frac{1}{r^3} \frac{d(r^4 \Omega^2)}{dr} > 0 \quad \text{i.e., stable}
\]
Papaloizou & Pringle Instability (‘84, 85)

The dynamical stability of differentially rotating discs with constant specific angular momentum

J. C. B. Papaloizou  Theoretical Astronomy Unit, School of Mathematical Sciences, Queen Mary College, London E1 4NS
J. E. Pringle  Institute of Astronomy, Madingley Road, Cambridge CB3 0HA

See also: Blases’85; Goldreich+’86; Kato’87; Narayan+’87; Hawley’90, etc.
Q: What happens if there is strong density gradients in disks?

\[ v_\varphi^2 = \frac{GM}{r} + \frac{r \partial p}{\Sigma \partial r} \]
Disk Basics

Rayleigh Criterion: \( \kappa^2 \equiv \frac{1}{r^3} \frac{d(r^4 \Omega^2)}{dr} > 0 \) i.e., stable

Solberg-Hoiland:

\[
\kappa^2(r) + N^2(r) \geq 0, \quad \text{where} \quad N^2 \equiv \frac{1}{\Sigma} \frac{d}{dr} \left( \frac{1}{\Sigma} \frac{d\Sigma}{dr} - \frac{1}{\Gamma P} \frac{dP}{dr} \right)
\]

p81:
Rayleigh’s inflexion point theorem:
extreme in potential vorticity profile
Learning from Geo/Planetary Communities …

SAO/NASA ADS Physics Abstract Service

- Find Similar Abstracts (with default settings below)
- Electronic Refereed Journal Article (HTML)
- References in the article
- Citations to the Article (1359) (Citation History)
- Refereed Citations to the Article
- Reads History

- Translate This Page

Title: On the use and significance of isentropic potential vorticity maps
Authors: Hoskins, B. J.; McIntyre, M. E.; Robertson, A. W.
Publication Date: 10/1985
Origin: WEB; CROSSREF
DOI: 10.1002/qj.49711147002
Bibliographic Code: 1985QJRMS.111..877H
Linear Theory Derivations (Lovelace et al. 1999; Li et al. 2000)

Potential Vorticity:

\[ F(r) \equiv \frac{\Sigma \Omega}{\kappa^2 - \Delta \omega^2 - c_s^2/(L_s L_p)} \]
\[ \mathcal{L} = \frac{\Sigma \Omega}{\kappa^2} (p \Sigma^{-\gamma})^{2/\gamma} \]

Rossby wave dispersion:

\[ \Delta \omega = -\frac{k \phi c_s^2/\Omega}{1 + k^2 h^2} \left[ (\ln \mathcal{L})' \pm \sqrt{[(\ln \mathcal{L})']^2 - \frac{1 + k^2 h^2}{L_s L_p}} \right], \]

(See also Tagger’01, Tsang & Lai’08, Meheut et al.’13, etc.)
Rossby Wave Instability

\[ \Psi''' + B(r)\Psi'' + C(r)\Psi = 0 \]

HGB case

Li et al. 2000
Rossby Vortices

Li et al. 2001

Pressure
Applications of RWI

1) RWI excited by the planet-disk interaction
2) RWI excited at the edge of “dead zone”
3) Effects of dust-gas coupling and feedback
From Birth to Maturity

Young disk

Transitional disk

HL Tau

HD 14252

Planets (no more disk)
Outstanding Issue #1: Accretion Physics

Armitage’ 11

Bai’ 16

Main parameters (set $\alpha=2\times10^{-4}$ at disk interior, 0.1 at MRI zone):
Outstanding Issue #2: Planet Formation

Mass Growth $\times 10^{33}$
Multiscale Problem

Global disk

Meso-scale vortex

Dust growth Streaming instability

This talk

$1/\text{size}$
Disks with “Rings”

Scenario A:
caused by planet-disk interaction
Disks with “Vortices”

Scenario B: caused by “deadzone”

Perez et al. ‘14
3D Equations for Gas+Dust

"Two-Fluid"

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]

\[
\frac{\partial (\rho u_r)}{\partial t} + \nabla \cdot (\rho \mathbf{u} u_r) = \rho \frac{u_r^2}{r} + \rho r \sin^2 \theta (\omega + \Omega)^2 - \frac{\partial p}{\partial r} - \rho \frac{\partial \Phi}{\partial r} + f_r
\]

\[
\frac{\partial \rho u_\theta}{\partial t} + \nabla \cdot (\rho \mathbf{u} u_\theta) = \rho r^2 \sin \theta \cos \theta (\omega + \Omega)^2 - \frac{\partial p}{\partial \theta} - \rho \frac{\partial \Phi}{\partial \theta} + r f_\theta
\]

\[
\frac{\partial \rho h_t}{\partial t} + \nabla \cdot (\rho h_t \mathbf{u}) = -\frac{\partial p}{\partial \phi} - \rho \frac{\partial \Phi}{\partial \phi} + r \sin \theta f_\phi
\]

\(f:\) include both viscous forces and dust-gas drag

\[
(\rho_d v_d)_t = -\rho_d \rho_g A (v_d - v_g) + \text{other terms}
\]

\[
(\rho_g v_g)_t = +\rho_d \rho_g A (v_d - v_g) + \text{other terms}
\]
Typical Simulation Setup

- 2D {r-phi} and 3D {r-phi-theta} geometry in LA-COMPASS
- Gas, dust, planets
- Self gravities of gas and dust are included
- Dust treated as pressureless fluid component
- Gas and dust coupled through drag force
- Mass of central star
- Planets
- Numerical resolution in 2D $N_r \times N_\phi = 6144 \times 6144$
  3D: $N_r \times N_{\theta} \times N_{\phi}: 512\times72\times768$
- Different dust particle sizes, different initial d/g ratio
- Disk sound speed profiles: $c_s(r)$
- Isothermal EOS
- Disk viscosity: $\alpha(r)$
Scenario A: RWI excited by Planet – Disk Interaction
Disk+planet Interaction (3D)

Li et al. 10 (unpublished)
3D Disk

10 M_{Earth}
Break the conservation of PV

\[
\frac{D(\zeta \hat{z})}{Dt} = (\zeta \hat{z} \cdot \nabla)v + \frac{1}{\Sigma} \nabla \Sigma \times \nabla p
\]

\[
\frac{D\zeta}{Dt} = \text{viscosity} + \text{shocks} + \text{non-adiabatic forcing}
\]

- Viscosity: either imposed or numerical
- Spiral shocks: cutting through the whole disk
- “Switch-on” of the planet

\[
\delta \zeta = \frac{1}{\Sigma_b} \frac{q^2}{1 + q} \frac{\partial Cr}{\partial S}
\]

Vorticity jump across shock: Truesdell (‘52), Lighthill (‘57), Hayes(‘57),…, Kevlahan (‘97)
Flow Lines, Horse-shoe Region and Shocks

Perpendicular Mach number

Distance away from planet

Li et al. 2005
Potential Vorticity Evolution

Li et al. 2005

Vorticity increase

Vorticity decrease

planet

Li et al. 2005
1 mm dust
d/g = 0.01
Without feedback

Fu, Li et al. 2014b
Dust+Gas: 1mm dust, no feedback

- Initial dust/gas density ratio = 0.01
- 1mm dust (St ~ 0.04)
- Time is in units of planet orbital period

Fu, Li et al. 2014b
1 mm dust
d/g = 0.01
With feedback

Fu, Li et al. 2014b
Vortices at Gap Edges

- Vortex forms at gap edge
- Dust feedback destroys vortex in a few thousand orbits
- Gap edge gradually becomes stable to RWI

Fu, Li et al. 2014
Dust vortex core mass evolution

Mass $/M_{\text{Earth}}$

Time (Number of planet orbits)

- $\rho_d/\rho_g = 0.001$
- $\rho_d/\rho_g = 0.002$
- $\rho_d/\rho_g = 0.003$
- $\rho_d/\rho_g = 0.01$
Modeling HL Tau Disk

( Jin, Li, Isella, et al. 2016)
Scenario B: RWI excited by disk “deadzone”

Accretion can drive mass pile up, leading to RWI
The dust size is 0.1mm, resolution is 512x72x768 in 
[0.4,4.448]x[1.2107,pi/2]x[0,2*pi] box
rho = rho_0*r^{-2.25}, T = T_0*r^{-0.5}. 

viscosity $\alpha$ distribution
$\Sigma r$ at 11500 turns
2D and 3D Produce Similar Results

\[ \rho r^{2.25} \text{ at } t=6400 \text{ orbits} \]

mid-plane density

\[ \Sigma r \text{ at } t=6400 \text{ orbits} \]

Surface density
Evolution of $m=1$ and $m=2$ modes in 2D vs. 3D

$m = 1$

$m = 2$
Synthetic Observations at 0.44 mm

Miranda, Li, et al.'16

[Diagram showing synthetic observations at 0.44 mm for different orbit counts.]
Summary

- Many disks show rings and non-axisymmetric features
- Giant planets –disk interaction and variable disk viscosity effects can produce these features
- Need much more work on coupling mesoscale (vortex) and small scales (dust growth)
- Need much more work on vortex-wave interactions in such disks