The Zombie Vortex Instability - a New, Fast, Robust Instability in Protoplanetary Disks

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Protoplanetary Disk thought to be Dead with respect to purely hydrodynamic instabilities

Rayleigh’s centrifugal stability theorem states that if the angular momentum (per unit mass) of a rotating fluid increases with radius, then the flow is centrifugally stable. This suggests that near-Keplerian flows are stable.
Flaws in assuming that PPDs are hydrodynamically stable (*dead*)

**Centrifugal Stability:** Rayleigh’s theorem applies only to a constant density fluid. A stratified, rotating fluid could be linearly unstable to a strato-rotational instability (SRI) (Le Dizes, Le Bars & Le Gal *PRL* 2007), or to a mode-coupling instability (Tevzadze et al. *A&A* 2008), or to a rotational instability (Le Dizes, Meunier, Riedinger),

The world is baroclinic, not always barotropic

Instabilities due to radial entropy gradients/vertical shear of the azimuthal velocity (Hubert Klahr)
Serendipitous Discovery of the Zombie Vortex Instability (ZVI)

Barranco & Marcus (2005, 2006): anelastic, shearing box, pseudospectral, vertical gravity linear in $|z|$, vertically isothermal so $N(z)$ linear in $|z|$

Initialized with an approximate 3D equilibrium vortex in the mid-plane with Rossby no. $Ro = 0.3$

Results after several hundred years: vortex linearly unstable, produces large amounts of inertio-gravity waves, unexplained patterns (unusual St Andrews crosses) and creates vortices off the mid-plane
Simplify to Understand the Physics

Equations of an annular section of a Protoplanetary Disk are similar to those of rotating, stratified, shearing Couette flow with a Boussinesq fluid

Use uniform vertical gravity $g$ and $N$

No dissipation other than hyperviscosity and hyperdiffusivity

Variety of boundary conditions

Single vortex or wave generator initial condition (Marcus et al PRL 2013)
Boussinesq Equations

\[
\frac{\partial \mathbf{v}}{\partial t} = - (\mathbf{v} \cdot \nabla) \mathbf{v} - \frac{1}{\rho_0} \nabla P + f \mathbf{v} \times \hat{\mathbf{z}} - \frac{(\rho - \rho_0) g}{\rho_0} \hat{\mathbf{z}}
\]

\[
\nabla \cdot \mathbf{v} = 0
\]

\[
\frac{\partial \rho}{\partial t} = - (\mathbf{v} \cdot \nabla) \rho
\]
Plane Couette Flow

\[ \bar{V}(y) \equiv -\sigma y \hat{x} \]
Plane Couette Flow

Shear – Cross-Stream \( y \)

\[
\bar{V}(y) \equiv -\sigma y \, \hat{x}
\]

Linearly Neutrally Stable
Plane Couette Flow with Rotation

\[ \vec{V}(y) \equiv -\sigma y \hat{x} \]

Rotation about the vertical \( z \) axis with \( f \equiv 2 \Omega \)

Linearly Neutrally Stable
Plane Couette Flow with Rotation and Stratification

\[ \bar{V}(y) \equiv -\sigma y \hat{x} \]

\[ N \equiv \sqrt{-\frac{g}{\rho_0} \frac{d\tilde{\rho}}{dz}} \]

Rotation about the vertical \( z \) axis with \( f \equiv 2 \Omega \)

Vertical density stratification of \( N(z) \)

neutrally stable if \( \sigma/f > -1 \)
Traditional Critical Layers

In unidirectional, dissipationless shear flows with $\vec{V}(y) \hat{x}$ at location $y^*$ where a neutrally stable eigenmode’s phase velocity $= \vec{V}(y^*)$, there is a traditional critical layer.

The eigenmode’s velocity in the $x$ direction is singular, but this eigenmode is difficult to excite. (Kelvin’s cats-eyes?)
Critical Layers

Undergraduate o.d.e. Explanation:

The location of the critical layer $y^*$ is where the coefficient of the highest derivative in the linearized stability equation (Rayleigh’s equation) vanishes.
New Critical Layers

In a unidirectional, dissipationless shear flow with $\bar{V}(y) \hat{x}$ with a stable vertically stratified density with Brunt-Vaisala frequency $N$, there is a new critical layer where the neutrally stable eigenmode’s velocity in the vertical $z$ direction is singular, and this eigenmode is important because it is very easy to excite.
Location of the critical layer $y^*$

\[ \bar{V}(y) - c_s \]

No density stratification in $z$

Traditional critical layer for horizontal shear in $y$
(Barotrophic critical layer)
Rayleigh equation

Extends to case of vertical shear rather than horizontal shear
Location of the critical layer $y^*$

$$
\begin{align*}
\left[ \bar{V}(y) - c_s \right] & \quad e^{i k_x (x - c_s t)} = e^{i (k_x x - s t)} \quad c_s \equiv s / k_x \\
\left[ \bar{V}(y) - \frac{s}{k_x} \right]
\end{align*}
$$
Location of the critical layer $y^*$

$$
\begin{align*}
\left[ \bar{V}(y) - c_s \right] e^{i k_x (x - c_s t)} &= e^{i(k_x x - st)} \\
\left[ \bar{V}(y) - \frac{s}{k_x} \right] \left[ \bar{V}(y) - \frac{s}{k_x} \pm \frac{N}{k_x} \right] & \quad \text{Add stratification in } z
\end{align*}
$$
Location of the critical layer $y^*$

\[ [\bar{V}(y) - c_s] e^{i k_x (x - c_s t)} = e^{i (k_x x - s t)} \quad c_s \equiv s / k_x \]

Add stratification in $z$

\[
\left[ \bar{V}(y) - \frac{s}{k_x} \right] \left[ \bar{V}(y) - \frac{s}{k_x} \pm \frac{N}{k_x} \right]
\]

\[ \bar{V}(y^*) - \frac{s}{k_x} \pm \frac{N}{k_x} = 0 \]
Location of the critical layer $y^*$

$$\left[ \bar{V}(y) - c_s \right] e^{i k_x (x - c_s t)} = e^{i (k_x x - st)} \quad c_s \equiv s / k_x$$

$$\left[ \bar{V}(y) - \frac{s}{k_x} \right]$$

Add stratification in $z$

$$\left[ \bar{V}(y) - \frac{s}{k_x} \right] \left[ \bar{V}(y) - \frac{s}{k_x} \pm \frac{N}{k_x} \right]$$

$$\bar{V}(y^*) - \frac{s}{k_x} \pm \frac{N}{k_x} = 0$$

$$\bar{V}(y) \equiv -\sigma y \hat{x}$$

$$y^* = - \frac{s \pm N}{\sigma k_x}$$
NEW Baroclinic Critical Layers

The new critical layers are easy to excite; the vertical component $v_z$ of velocity eigenmode is singular; and the layers appear as vortex layers aligned with the stream-wise axis.

\[
\frac{\partial \omega_z}{\partial t} = -(v \cdot \nabla) \left( \omega_z + \frac{d \bar{V}}{dy} \right) + (\omega \cdot \nabla)v_z + \left( f - \frac{d \bar{V}}{dy} \right) \frac{\partial v_z}{\partial z}
\]

Vorticity equation

Derived from the $z$ component of the momentum equation.
Energetics

The energy that drives the vortices is not supplied by the original vortex or by the wave generator.

The energy comes from the differential motion of the background shear flow.
NEW Baroclinic Critical Layers

\[ y^* = -\frac{s L}{2\pi m \sigma} \pm \frac{L N}{2\pi m \sigma} \]

unit of length = \((LN)/(2\pi \sigma)\)
unit of time = \(1/N\)

\[ y^* = -\frac{s \pm 1}{m} \]

\(y^*\) is the distance from a perturbation to the critical layer; \(\Delta \equiv \frac{L N}{2\pi \sigma}\)
\(s\) is the frequency of the perturbation
For a perturbation from a steady vortex, $s = 0$

\begin{align*}
y^* &= \frac{s \pm 1}{m} \\
|y^*| &< 1
\end{align*}
Plane Couette Flow

\[ \bar{V}(y) \equiv -\sigma y \hat{x} \]

Shear - Cross-Stream $y$

Stream-wise $x$
For \( s = 0 \)

\[
y^* = \pm \frac{1}{m}
\]

\[
|y^*| < 1
\]

The flow is invariant under translation in \( x, z \), and a translation in \( y \) accompanied by a Galilean boost in velocity.

Each new vortex that is created acts like a new, self-similar perturbation.
The Instability Derives Its Energy from the Shear

\[ \omega_z \text{ at } x-y \text{ plane } z=0.68638 \ t=0 \]
Perturbation of 1 vortex – Boussinesq $N$ constant

Vertical $z$

Shear – Cross-Stream $y$
Steps of Zombification

Energize the critical layer with a finite amplitude trigger - trigger depends on vorticity, not velocity

Vortex stretching turns critical layer into braided vortex layers

Vortex layer with vorticity the same sign as ambient shear is linearly unstable (identical to GRS (Nature 1988) a.k.a. the Rossby instability in PPD

Roll-up of vortex layer into discrete vortices with $|\text{Ro}| > \sim 0.2$ (JFM 1990)

Those vortices trigger instability in neighboring critical layer

Lattice of vortices becomes space-filling turbulence
Properties of Critical Layers

We (Patrick Huerre and Meng Wang) understand:

The order of the singularity and how the higher order terms (viscosity thermal diffusivity) remove the singularities

The thickness of a linear critical layer, \( \delta \sim \left[\frac{\nu}{(\sigma k_x)}\right]^{1/3} \) & jump in velocity \( \Delta v \)

Spacing between critical layers is \( \frac{N(z)}{(\sigma k_x)} \)

Thin critical layer possibly makes dissipation important

\( \text{Re} \equiv \frac{\sigma \delta^2}{\nu} > \sim 35, \text{Pe} \equiv \frac{\sigma H^2}{k} > \sim 5 \times 10^4 \)

Do not understand: details of the finite-amplitude trigger, or thickness of the nonlinear critical layer
Robust?

Boundaries:

cross-stream – shearing box, periodic with saw-tooth shear, no normal component of the velocity, no-slip boundaries

Vertical – periodic, “saw tooth”, no normal component of the velocity, mapped to with Ferziger functions

$g N$: Uniform or linear $g$; uniform, linear or complex $N(z)$

Eq. of state: Boussinesq, anelastic fully-compressible

Dissipation: hyperviscosity, hyperdiffusivity Newton cooling, thermal diffusivity, molecular viscosity

Scheme: pseudospectral, spectral, finite volume
Initial Conditions: Noise

• Instability of disk: initial condition: Keplerian flow + noise

\[ E_k(k) \sim k^{-a} \]

\[ \nu_{rms}(k) \sim k^{(1-a)/2} \quad \nu_{rms}(l) \sim \nu_{rms}(L) \left( \frac{l}{L} \right)^{-(1-a)/2} \]

\[ \omega_{rms}(k) \sim k^{(3-a)/2} \quad \omega_{rms}(l) \sim \omega_{rms}(L) \left( \frac{l}{L} \right)^{-(3-a)/2} \]

For \( 1 < a < 3 \), \( \nu_{rms} \) is at the largest scale

\( \omega_{rms} \) is at the smallest scale

Kolmogorov: \( 1 < a = 5/3 < 3 \)
Rossby Number

$\omega(k)/f \sim k^{2/3}$

Initial Mach number = 0.0021

Kolmogorov Spectrum
No Zombies

Resolution
Initial Mach number = 0.0028

Rossby Number

$\omega(k)/f \sim k^{2/3}$

Kolmogorov Spectrum

Zombies
Initial Mach number = 0.002

Steeper than Kolmogorov Spectrum
Zombies!!!
Rossby Number

\[ \frac{\omega(k)}{f} \sim k^{2/3} \]

Initial Mach number = 0.0021

Kolmogorov Spectrum

Zombies!!!!
Rossby Number

\( \omega(k)/f \approx k^{2/3} \)

Initial Mach number = 0.0021

\( M_{\text{critical}} > R e^{-1/2} \) (PSM et al 2016)

Kolmogorov Spectrum

Zombies!!!!
streamwise/cross-stream
Robust with Respect to Dissipation

Computed with Newton cooling times:
“Zombifies” for cooling times greater than 2.5 years – due to thin critical layer
$\sim 10^{-5} - 10^{-3} \, H$ – optically thin

Lesur & Latter (2016) claim the disk mid-plane cooling time of $10^{-3}$ kills zombies*

*For Boussinesq flows with uniform $g, N$, and cooling times
Vertical $g$ is linear in $|z|$, and $N(z)$ is nearly so (models (Dullemond & Dominik (2004) and D’Alessio et al. (2006))

Cooling time (optically thin) is approximately inversely proportional to dust density

If dust settles quickly, $10^{4-5}$ years (Garaud et al. 2004) and is re-lofted (Barranco (2009); Chiang (2008); Lee et al. (2010a,b)) to have a Gaussian distribution about the mid-plane with a vertical scale height less than 0.1 H, and the cooling time at the mid-plane is $10^{-3}$ years, then the cooling time at $z = H$ is much longer.

Onset requires $\sim 2 > N/f > \sim 1$, so in a disk expect growth to begin between 1 and 2 $H$ above/below the mid-plane.
Profiles motivated by models (Dullemond & Dominik (2004) and D’Alessio et al. (2006))
Cooling times based on optically thin dust-filled hydrogen w/ $10^{-3}$ years at mid-plane
Vertical $z/H$
Zombie Turbulence is Turbulence

Kolmogorov spectrum at all but the largest scales

Large anticyclonic vortices and cyclonic sheets have long coherence times, but not the small scales

Attracting solutions: Zonal flows with late time “oscillations” 10% of Keplerian velocity

Mach and Rossby numbers of order unity

Robust with respect dissipation

ZVI is not just an already known shear instability; it is highly anisotropic and intermittent with large features
**“To Do” List**

Laboratory Couette -Taylor flow with co-rotating inner & outer cylinders & vertical stratification with salt

Effects on dust accumulation and planetesimal formation

Now have a working spectral fully-compressible code almost as fast as the anelastic one and with a time-step nearly as large: acoustic waves and angular momentum transport (acoustic waves have correlated $x$ and $y$ components)

Are there direct observables? Chondrules (Cuzzi et al. 1993) James Webb (timescales, high/low turbulent states)?