Solar Dynamo with Spot Deposition

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Solar Cycle: The cyclic variation of the number of sunspots.
Butterfly diagram of sunspot

Latitudinal positions of sunspots in time.
(Mandal et al. 2016)

- Why it is 11 years?
- Why sunspot propagate equatorward?
- Why surface radial field propagate poleward?

Background color: the radial magnetic field on the solar surface

Image: D. Hathaway/NASA
Mathematical equations in dynamo process

Induction Eq.: \[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \nabla \times \mathbf{B}), \quad \text{with } \nabla \cdot \mathbf{B} = 0
\]

Momentum Eq.: \[
\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B} - 2\rho \Omega_0 \times \mathbf{v} + 2 \nabla \cdot \nu \rho \mathbf{S},
\]

Continuity Eq.: \[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{V},
\]

Energy Eq.: \[
\rho T \frac{D\mathbf{s}}{Dt} = \nabla \cdot (K \nabla T) - \rho^2 Q(T) + 2\rho \nu \mathbf{S}^2 + \frac{\mathbf{J}^2}{\sigma}.
\]

Equation of state and proper boundary conditions.

== Direct numerical simulation (DNS) very challenging!
(because of large spatial and temporal scale and smaller values of fluid and magnetic diffusivity in the solar convection zone.)
Results from convection simulations

Karak et al. (2015)
Dynamo models

\[ \frac{\partial B}{\partial t} = \nabla \times (V \times B - \eta \nabla \times B), \quad \text{with} \quad \nabla \cdot B = 0 \]

\[ \rho \frac{Dv}{Dt} = -\nabla p + \rho g + J \times B - 2\rho \Omega_0 \times v + 2\nabla \cdot \nu \rho S, \]

\[ \frac{D\rho}{Dt} = -\rho \nabla \cdot V, \]

\[ \rho T \frac{Ds}{Dt} = \nabla \cdot (K \nabla T) - \rho^2 Q(T) + 2\rho \nu S^2 + \frac{J^2}{\sigma}. \]

Kinematic mean-field model

– little easy, but no dynamics!
Mean-field dynamo model
(Parker 1955; Steenbeck, Krause & Raedler 1966)

\[ \mathbf{v} = \bar{\mathbf{v}} + \mathbf{v}', \quad \mathbf{B} = \bar{\mathbf{B}} + \mathbf{B}', \text{ with } \bar{\mathbf{B}}' = 0 \text{ and } \bar{\mathbf{v}}' = 0 \]

Mean-field induction equation:

\[
\frac{\partial \bar{\mathbf{B}}}{\partial t} = \nabla \times (\bar{\mathbf{V}} \times \bar{\mathbf{B}} + \mathbf{\varepsilon} - \eta \nabla \times \bar{\mathbf{B}})
\]

Where

\[ \varepsilon = \mathbf{v}' \times \bar{\mathbf{B}}' \]

After approximation:

\[ \varepsilon = \alpha \bar{\mathbf{B}} - \beta \nabla \times \bar{\mathbf{B}} \]

where

\[
\begin{align*}
\alpha &= -\frac{1}{3} \frac{1}{\mathbf{v}' \cdot (\nabla \times \mathbf{v}')} \\
\beta &= \frac{1}{3} \frac{1}{\mathbf{v}' \cdot \mathbf{v}' \tau}
\end{align*}
\]
Large-scale global dynamo: the basic idea

(Parker 1955; Steenbeck, Krause & Raedler 1966; Babcock 1961; Leighton 1969)

Observationally verified!
Equatorward migration of sunspots

Dynamo wave?
(Parker–Yoshimura sign rule
1955, 1975; Stix 1976)

Positive alpha and negative radial shear give equatorward propagation.
Equatorward migration of sunspots

Dynamo wave?
(Parker–Yoshimura sign rule 1955, 1975; Stix 1976)
Positive alpha and negative radial shear give equatorward propagation!

Flux transport dynamo
(Wang, Sheeley & Nash 1991; Durney 1995; Choudhuri, Schussler & Dikpati 1995, and many more)

Period $\propto \frac{1}{\nu_0^{0.89}}$
(Dikpati & Charbonneau 1999; Yeates, Nandy & Mackey 2008)
Mixing length theory $\sim 10^{13} \text{ cm}^2/\text{s}$

Karak (2010); Karak & Choudhuri (2010): $\sim 10^{12} \text{ cm}^2/\text{s}$

Miesch et al (2012): $10^{12} \text{ cm}^2/\text{s}$

Cameron & Schussler (2016): $3 \times 10^{12} \text{ cm}^2/\text{s}$ (Simard et al. 2016)

Can quenching help? (Rudiger, Kitchatinov & Pipin 1994)

Figure from Munoz-Jaramillo et al. (2011)
Quenching of the diffusivity with $B$

Measured from convection simulations -- Karak et al. (2014)

Confirmed by Simard et al. (2016) in a more realistic simulation.

\[
\eta = \frac{\eta_0}{1 + \left(\frac{B}{B_{eq}}\right)^{1.3}}
\]
Constructing a high diffusivity dynamo model

\[
\frac{\partial B}{\partial t} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (rv_r B) + \frac{\partial}{\partial \theta} (v_\theta B) \right] = \eta_t (\nabla^2 - \frac{1}{s^2}) B + s (B_p \cdot \nabla) \Omega
\]

\[
\frac{\partial A}{\partial t} + \frac{1}{s} (v \cdot \nabla)(sA) = \eta_t (\nabla^2 - \frac{1}{s^2}) A + \alpha B
\]
Increased alpha coefficient!

=> Consistent with $\alpha \Omega$ dynamo wave frequency (Stix 1976; Koehler 1973)

\[\alpha_0 = 1.3 \text{ m s}^{-1}\]

\[
\begin{array}{c}
\text{Latitude} \\
-90 & -60 & -30 & 0 & 30 & 60 & 90
\end{array}
\]

\[
\begin{array}{c}
0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18
\end{array}
\]

\[
\begin{array}{c}
\text{Time (years)} \\
0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18
\end{array}
\]

Figure 16. Time-latitude diagram of the toroidal magnetic field (in Gauss) integrated over the whole CZ for the simulation with $\nu_f = 0$ (no pumping) and $\alpha_0 = 18.0 \text{ m s}^{-1}$. Period = 2.4 year
For $\eta = 1.5 \times 10^{12} \text{ cm}^2/\text{s}$: 
Diffusion time from bottom to surface $\sim 6$ years.

So the cycle period must be less than 6 years!
Turbulent electromotive force in mean-field dynamo

\[ \mathcal{E} = \alpha \mathbf{B} - \eta_t \nabla \times \mathbf{B} \]

\[ \mathcal{E} = (\alpha \mathbf{B} + \gamma \times \mathbf{B}) - \eta_t \nabla \times \mathbf{B} \]

Pumping of magnetic flux

Density pumping (due to a gradient in \( \rho \)),
Turbulent pumping (gradient of velocity),
Topological pumping (topological asymmetry of the flow),
etc.
Turbulent electromotive force in mean-field dynamo

\[ \mathcal{E} = \alpha B - \eta_t \nabla \times B \]

Convection simulations:
Pumping speed \(~ 1/10th\) of convective velocity (Kapyla et al. 2009; Karak et al. 2015; Warnecke et al. 2016...)
Simulation with 35 m/s downward pumping in top 10% of solar radius
Dynamo solution

Top: Radial field at surface

Bottom: mean toroidal field over the whole convection zone.

Equatorward migration is caused by dynamo wave + flow
3D Solar Dynamo with Spot Deposition

\[
\frac{\partial B}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})
\]

- At present, the model is kinematic
- Diffusivity: $2-4 \times 10^{12}$ cm$^2$/s in the whole convection zone.
- In the near-surface layer, we have a downward magnetic pumping of about 20 m/s.

We give a tilt to each sunspot given by Joys law:

\[
\delta = \frac{32.1^\circ}{1 + (B/B_{eq})^2} \cos(\theta)
\]

(Stenflo & Kosovichov 2012; Wang et al. 2015)
We do not put spot at every integration time step! Rather we take it from a log-normal distribution such that we have frequent eruptions when magnetic field is stronger.

\[
P(\Delta) = \frac{1}{\sigma \Delta \sqrt{2\pi}} \exp \left[ -\frac{(\ln \Delta - \mu)^2}{2\sigma^2} \right]
\]

\[
\sigma^2 = \left(\frac{2}{3}\right) [\ln(\tau_s) - \ln(\tau_p)]
\]

\[
\mu = \ln \tau_p + \sigma^2.
\]

**Median:**

\[
\tau_p = \frac{1}{1 + \frac{Ben^N}{B^2_\tau}} \text{ days},
\]

**Mean:**

\[
\tau_s = \frac{12}{1 + \frac{Ben^N}{B^2_\tau}} \text{ days},
\]

This is how we couple the toroidal field at the bottom of the convection zone to the flux in sunspots.

We note that the time delay is computed separately in each hemisphere!
A steady dynamo solution with tilt angle quenching

Evolution of the mean toroidal and poloidal magnetic fields

Evolution of the radial magnetic field on the surface.

Observed flux
A steady dynamo solution with weak angle quenching

Observed flux

Evolution of the radial magnetic field on the surface.

Evolution of the mean toroidal and poloidal magnetic fields
Dynamo solution with tilt fluctuations consistent with observations!
Dynamo solution with four times the observed fluctuations.
Dynamo solution with four modes the observed fluctuations.
Dynamo solution with four times the observed tilt fluctuations.
Conclusions

1. We are developing a 3D dynamo model, we call it STABLE (Surface flux Transport and Babcock-Leighton) dynamo model.

2. This model is driven by the tilted bipolar sunspots with properties taken from observations.

3. This model is robust under large variations around the Joy’s law.

Thank you
Conclusions

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Thank you