Simulations and understanding large-scale dynamos

- Issues with global models
- Possibility of smaller scales
- Consequences of this
- Magnetic flux concentrations
- Unusual dynamo effects

Axel Brandenburg
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Global models suggest

- Distributed dynamo action
  - Difference to flux transport dynamos
  - Would require smaller turb. diff.
    \[ \eta_t = \frac{u_{rms}}{3k_f} = \frac{u_{rms}}{\ell/3} \]

- Surface flux from upper layers
  - Difference to deeply rooted tube picture
  - Surface flux reamplification needed
    - NEMPI: works best for large \( k_fH_p \)

- Mostly cylindrical \( \Omega \)-contours
  - Anti-solar differential rotation
Anomalously weak solar convection

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Contributed by Katepalli R. Sreenivasan, May 3, 2012 (sent for review December 30, 2011)

Convection in the solar interior is thought to comprise structure on a spectrum of scales. This conclusion emerges from phenomenological studies and numerical simulations, though neither covers the proper range of dynamical parameters of solar convection. Here, we analyze observations of the wavefield in the solar photosphere using techniques of time-distance helioseismology to image flows in the solar interior. We downsample and synthesize 9000 radial-harmonic degree $\ell$. Within the wavenumber band $\ell < 60$, convective velocities are 20–100 times weaker than current theoretical estimates. This constraint suggests the prevalence of a different paradigm of turbulence from that predicted by existing models, prompting the question: what mechanism transports the heat flux of a solar luminosity outwards? Advection is dominated by Coriolis...
Do we need to rethink?

• In mixing length theory: $l=H_p$ only hypothesis
  – cf. Nick Featherstone’s talk
• Simulations: subgrid scale diffusion, viscosity
• Envisage reasons for (i) smaller scale flows and/or (ii) deeper parts subadiabatic?
• But depth of convection zone still 200 Mm
CONVECTION IN STELLAR ENVELOPES: A CHANGING PARADIGM

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ABSTRACT. Progress in the theory of stellar convection over the past decade is reviewed. The similarities and differences between convection in stellar envelopes and laboratory convection at high Rayleigh numbers are discussed. Direct numerical simulation of the solar surface layers, with no other input than atomic physics, the equations of hydrodynamics and radiative transfer is now capable of reproducing the observed heat flux, convection velocities, granulation patterns and line profiles with remarkably accuracy. These results show that convection in stellar envelopes is an essentially non-local process, being driven by cooling at the surface. This differs distinctly from the traditional view of stellar convection in terms of local concepts such as cascades of eddies in a mean superadiabatic gradient. The consequences this has for our physical picture of processes in the convective envelope are illustrated with the problems of sunspot heat flux blocking, the eruption of magnetic flux from the base of the convection zone, and the Lithium depletion problem.
Entropy rain

The first consequence of the extreme asymmetry between top and bottom is that upward moving fluid is isentropic, hence neutrally buoyant, all the way from the base of the convection zone to the thermal boundary layer at the top. It follows that we cannot say anymore that the flow is driven by buoyant bubbles moving up from below. All driving (in terms of actual forces) is due to cooling at the surface. The gently rising isentropic upflow gets exposed to the cold universe in a thin layer near optical depth unity. The extreme temperature dependence of the opacity makes the cooling happen even faster than it would have done otherwise: as the fluid cools, it becomes more transparent, and cools even faster. The cooled fluid has strong (negative) buoyancy, and collects in a downward flowing lane. On its way down the lanes very soon break up into threads or ‘raindrops’. The upward flow of the hot fluid serves to replace the fluid ‘condensing’ at the top. The entropy contrast of the downflows, the horizontal and vertical velocities, the length scales of the upwellings are all determined by the physics happening in the surface boundary layer.
Stein & Nordlund (1998) simulations

Filamentary, nonlocal shown: entropy fluctuations pos neg
Tau approximation

\[ \dot{s} = -u_j \nabla_j \bar{S} + N_s \]

\[ \dot{u}_i = g_i s / c_p + N_u \]

\[ \frac{\partial F_i}{\partial t} \propto u_i \dot{s} + \dot{u}_i s = -u_i u_j \nabla_j \bar{S} + g_i s^2 / c_p + N_{su} \]

Closure hypothesis

\[ N_{su} = - \frac{F_i}{\tau} \]
Theoretical Expression for the Countergradient Vertical Heat Flux

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A theoretical expression is derived from the heat-flux conservation equation for the counter potential-temperature gradient that can sustain an upward flux of sensible heat. This gradient is found to be \( \gamma_c = \left( g/\theta \right) \left( \theta'^2 \right)/\langle w'^2 \rangle \), where \( \langle \theta'^2 \rangle \) is the potential temperature variance and \( \langle w'^2 \rangle \) is the vertical velocity variance. The usual down-gradient eddy coefficient expression for the heat flux is obtained from the derivation only if \( \gamma_c \) is set to zero. Aircraft measurements of \( \left( g/\theta \right) \left( \theta'^2 \right)/\langle w'^2 \rangle \) in the middle and upper portions of convective planetary boundary layers indicate that this expression for \( \gamma_c \) is of the same order of magnitude (near \( 0.7 \times 10^{-5} \, {\text{K cm}}^{-1} \)) as the value deduced previously for \( \gamma_c \) from completely different considerations.

Evidence has been accumulating for many years that in the central half or so of the planetary boundary layer (PBL) under conditions of upward sensible heat flux the lapse rate is slightly less than adiabatic; that is, the heat flux is countergradient. The evidence up to 1966 is summarized by Deardorff [1966]. Since that time, additional aircraft observations of both potential temperature and heat flux by Lenschow [1970] and by Warner [1971] demonstrate that could represent a much larger averaging area or a time or ensemble average. Although this value of \( \gamma_c \) is small, the question of its use is far from being of only academic concern. In a thick PBL of 3 km height, for example, a numerical model that uses equations 1 and 2 will predict values of \( \left( \theta \right) \) at upper levels in the PBL (relative to lower levels) nearly 2 K warmer than one that sets \( \gamma_c = 0 \) for a given set of external conditions.
and \( \gamma \).

Deardorff [1972], Wyngaard et al. [1971], and Donaldson [1972] that utilize equations for the second moments and closure assumptions for third moments. The equation, which makes use of the Boussinesq approximation, is

\[
\frac{\partial}{\partial t} \langle w' \theta' \rangle = -\langle u_i \rangle \frac{\partial}{\partial x_i} \langle w' \theta' \rangle - \langle w'u_i' \rangle \frac{\partial \langle \theta \rangle}{\partial x_i} \\
- \langle u_i' \theta' \rangle \frac{\partial \langle w \rangle}{\partial x_i} - \frac{\partial}{\partial x_i} \langle w'u_i' \theta' \rangle \\
+ \frac{g}{\theta_0} \langle \theta'^2 \rangle - \frac{1}{\rho_0} \langle \theta' \frac{\partial p'}{\partial z} \rangle
\]

(3)
Physical meaning?

\[ \frac{s}{c_p} = \frac{1}{\gamma} \ln p - \ln \rho \]

\[ s > 0, \quad u_z > 0 \implies u_z s > 0 \]
Physical meaning?

\[ \frac{s}{c_p} = \frac{1}{\gamma} \ln p - \ln \rho \]

\[ s < 0, \quad u_z < 0 \quad \Rightarrow \quad u_z s > 0 \]
Why should only the top be unstable

\[ F_{\text{rad}} = -K \frac{dT}{dz} = \text{const} \]

e.g. if \[ \frac{dT}{dz} = \text{const} \]

\[ K = \frac{16\sigma T^3}{3\kappa\rho} = \text{const} \]

Power law \[ \kappa = \kappa_0 \rho^a T^b \]

Polytropic index \( n \) \[ \rho = T^{\frac{3-b}{1+a}} = T^n \]
Deeper parts intrinsically stable

Kramers opacity (interior): $a=1$, $b=-7/2$

Polytropic index $n$ \[ \rho = T^{\frac{3-b}{1+a}} = T^n \]

Entropy gradient positive (stable) for $n > 3/2$
Solar opacities

\[ n \ll -1 \quad \text{and} \quad n = 3.25 \]
Hydrostatic reference solutions

Double Kramers-like

$$K^{-1} = K_{H^{-}}^{-1} + K_{Kr}^{-1}$$

Thickness only

$\sim 1\text{Mm}$
On the Convection of the Stellar Photospheres*

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(Received May 4, 1950)

In these earlier works, the opacity of the stellar photospheres has been attributed to the continuous absorption of neutral hydrogen for the early type stars and to that of various metallic atoms for the late type stars. But, since 1938 it has been pointed out by R. Wildt(6) that for the late type stars the opacity of negative hydrogen ion instead of metals plays a leading role in the mechanism of continuous absorption, the effect on the photospheric structure has become to be considered as relevant, as has been shown by R. Wildt(7) and B. Strömgren(8). Several years later, the coefficient of continuous absorptions of H− has been in detail computed by Chandrasekhar and his collaborators(9). Consequently it may be of interest to revise the theory of convection allowing for the continuous absorptions of atoms and negative ions of hydrogen.
Original mixing length model

\[ F_{\text{conv}} = -\frac{1}{3} \tau u_{\text{rms}} \nabla \bar{S} \]
New solutions with Deardorff flux

Entropy gradient

$$\nabla = \frac{d \ln T}{d \ln p}$$

$$-d(\bar{S} / c_p) / dz = (\nabla - \nabla_{\text{ad}}) / H_p$$

old

$$F_{\text{conv}} \propto (\nabla - \nabla_{\text{ad}})$$

$\rightarrow$ new

$$F_{\text{conv}} \propto (\nabla - \nabla_{\text{ad}}) + \nabla_D$$

arXiv:1504.03189v2
Consequences of small scales

- Larger $k_f \rightarrow$ less turb. Diffusion: $\eta_t = \frac{u_{\text{rms}}}{3k_f}$
- Applications to dynamos: stronger, less turb diffusive
  - Helps flux transport dynamos
- Two other important effect:
  - Lambda effect $\rightarrow$ differential rotation (Co smaller, Ta larger)
  - Baroclinic term stronger?
  - Negative effective magnetic pressure $\rightarrow$ spots
Flux emergence in global simulations

Nelson, Brown, Brun, Miesch, Toomre (2014)
3 scenarios

• Rising flux tubes?
• Hierarchical convection?
• Self-organization as part of the dynamo

\[ g.B \rightarrow u.B \quad k \, H_p \]

\[ g.\Omega \rightarrow u.\omega \rightarrow A.B \]
ON THE TURBULENT DECAY OF STRONG MAGNETIC FIELDS AND THE DEVELOPMENT OF SUNSPOT AREAS

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(Received 29 October, 1974; in revised form 24 January, 1975)

Sunspot decay as a test of the eta-quenching concept

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Self-assembly of a magnetic spot

- Minimalistic model
- 2 ingredients:
  - Stratification & turbulence
- Extensions
  - Coupled to dynamo
  - Compete with rotation
  - Radiation/ionization
A possible mechanism

\[ U_i U_j - B_i B_j + \frac{1}{2} \delta_{ij} B^2 = \frac{1}{3} \delta_{ij} \left( U^2 + \frac{1}{2} B^2 \right) \approx \frac{1}{3} \delta_{ij} \left( U^2 + B^2 - \frac{1}{2} B^2 \right) \approx \text{const} \]

**Re}_M here based on forcing \( k \)

Here 15 eddies per box scale

\( \text{Re}_M = 70 \) means \( 70 \times 15 \times 2\pi = 7000 \) based on box scale


Breakdown of quasi-linear theory
Negative effective magnetic pressure instability

Kleeorin, Rogachevskii, Ruzmaikin (1989, 1990)

- Gas+turbulent+magnetic pressure; in pressure equil.
- \(B\) increases \(\rightarrow\) turbulence is suppressed
- \(\rightarrow\) turbulent pressure decreases
- Net effect?

\[t = 1100.0\]
**Sunspot formation that sucks**

Mean-field simulation: Neg pressure parameterized

Typical downflow speeds \( \text{Ma}=0.2 \ldots 0.3 \)

Brandenbur et al (2014)
Bi-polar regions in simulations with corona

Warnecke et al. (2013, ApJL 777, L37)
Coronal loops?

Warnecke et al. (2013, ApJL 777, L37)
First dynamo-generated bi-polar regions

Mitra et al. (2014, arXiv)
Still negative effective magnetic pressure? Or something new?

Mitra et al. (2014, arXiv)
Global models

Jabbari et al. (2015, arXiv)
New aspects in mean-field concept

Ohm’s law
\[ \eta \mathbf{J} = \mathbf{E} + \mathbf{U} \times \mathbf{B} + \mathbf{u} \times \mathbf{b} \]

Theory and simulations: \( \alpha \) effect and turbulent diffusivity
\[ \mathbf{u} \times \mathbf{b} = \alpha \mathbf{B} - \eta_t \mathbf{J} + \ldots \]

Turbulent viscosity and other effects in momentum equation
\[ \mathbf{u}_i \mathbf{u}_j = \ldots - \nu_t ( \mathbf{U}_{i,j} + \mathbf{U}_{j,i} ) + q_s \mathbf{B}_i \mathbf{B}_j - \frac{1}{2} \delta_{ij} q_p \mathbf{B}^2 + \ldots \]
Calculate full $\alpha_{ij}$ and $\eta_{ij}$ tensors

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} + \mathbf{E} - \eta \mathbf{J}$$

**turbulent emf**

$$\mathbf{E} = \mathbf{u} \times \mathbf{b}$$

**$\alpha$ effect and turbulent magnetic diffusivity**

$$\mathbf{E}_j = \alpha_{ij} \mathbf{B}_j - \eta^*_{ij} \mathbf{J}_j$$

- **Imposed-field method**
  - Convection (Brandenburg et al. 1990)
- **Correlation method**
  - MRI accretion discs (Brandenburg & Sokoloff 2002)
  - Galactic turbulence (Kowal et al. 2005, 2006)
- **Test field method**
  - Stationary geodynamo (Schrinner et al. 2005, 2007)
Calculate full $\alpha_{ij}$ and $\eta_{ij}$ tensors

\[
\frac{\partial A}{\partial t} = U \times B - \eta J
\]
Original equation (uncurled)

\[
\frac{\partial \bar{A}}{\partial t} = \bar{U} \times \bar{B} + \bar{u} \times \bar{b} - \bar{\eta} J
\]
Mean-field equation

\[
\frac{\partial a}{\partial t} = \bar{U} \times b + \bar{u} \times \bar{B} + \bar{u} \times b - \bar{u} \times b - \eta j
\]
fluctuations

Response to arbitrary mean fields

\[
\frac{\partial a^{pq}}{\partial t} = \bar{U} \times b^{pq} + \bar{u} \times \bar{B}^{pq} + \bar{u} \times b^{pq} - \bar{u} \times b^{pq} - \eta j^{pq}
\]
**Test fields**

$$\mathbf{\bar{B}}^{11} = \begin{pmatrix} \cos k_z \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{\bar{B}}^{21} = \begin{pmatrix} \sin k_z \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{\bar{B}}^{12} = \begin{pmatrix} 0 \\ \cos k_z \\ 0 \end{pmatrix}, \quad \mathbf{\bar{B}}^{22} = \begin{pmatrix} 0 \\ \sin k_z \\ 0 \end{pmatrix}$$

$$\mathbf{\bar{E}}^{pq} = \alpha_{ij} \mathbf{\bar{B}}_{j}^{pq} + \eta_{ijk} \mathbf{\bar{B}}_{j,k}^{pq}$$

**Example:**

$$\mathbf{\bar{E}}_{1}^{11} = \alpha_{11} \cos k_z - \eta_{113} k \sin k_z$$

$$\mathbf{\bar{E}}_{1}^{21} = \alpha_{11} \sin k_z + \eta_{113} k \cos k_z$$

$$\begin{pmatrix} \alpha_{11} \\ \eta_{113} k \end{pmatrix} = \begin{pmatrix} \cos k_z & \sin k_z \\ -\sin k_z & \cos k_z \end{pmatrix} \begin{pmatrix} \mathbf{\bar{E}}_{1}^{11} \\ \mathbf{\bar{E}}_{1}^{21} \end{pmatrix}$$

$$\begin{pmatrix} \eta_{11}^* \\ \eta_{12}^* \\ \eta_{21}^* \\ \eta_{22}^* \end{pmatrix} = \begin{pmatrix} \eta_{123} \\ -\eta_{113} \\ \eta_{223} \\ -\eta_{213} \end{pmatrix}$$
Kinematic $\alpha$ and $\eta_t$ independent of $R_m$ (2...200)

\[ \alpha_0 = -\frac{1}{3} \tau \langle \omega \cdot u \rangle \]
\[ \eta_0 = \frac{1}{3} \tau \langle u^2 \rangle \]
\[ \tau = \left( u_{\text{rms}} k_f \right)^{-1} \]
\[ \alpha_0 = -\frac{1}{3} u_{\text{rms}} \]
\[ \eta_0 = \frac{1}{3} u_{\text{rms}} k_f^{-1} \]

Nonlocality: convolution

\[ \mathcal{E}_i(z, t) = \iiint (\alpha_{ij}(\zeta, \tau) \overline{B}_j(z - \zeta, t - \tau) \]
\[ - \eta_{ij}(\zeta, \tau) \mu \overline{J}_j(z - \zeta, t - \tau)) \, d\zeta \, d\tau \]

- Multiplication \(\rightarrow\) convolution
- Babcock-Leighton effect is an example
- Sharp structures in mean-field dynamos artifacts
- Convolution in \(x\)-space \(\rightarrow\) multiplication in \(k\)
The 4 Roberts flows

\[ u_x = v_0 \sin k_0 x \cos k_0 y, \quad u_y = -v_0 \cos k_0 x \sin k_0 y, \]

while the components \( u_z \) are different and given by

\[ u_z = w_0 \sin k_0 x \sin k_0 y \quad \text{(flow I)}, \]
\[ u_z = w_0 \cos k_0 x \cos k_0 y \quad \text{(flow II)}, \]
\[ u_z = \frac{1}{2} w_0 (\cos 2k_0 x + \cos 2k_0 y) \quad \text{(flow III)}, \]
\[ u_z = w_0 \sin k_0 x \quad \text{(flow IV)}, \]

IV flow: negative eddy diffusivity dynamo

→ But positive diffusion at small scales

Devlen et al. (2013)
Time-delay dynamo for Roberts II and III flows

\[ \partial_t B_x = -\gamma \partial_z B_x + \eta \partial_z^2 B_x \]

\[ \lambda = -ik\gamma + \eta k^2 \quad \text{decay oscillatory} \]

With time delay

\[ \partial_t B_x = -\gamma \partial_z B_x (t - \tau) + \eta \partial_z^2 B_x \]

\[ \partial_t B_x = -\gamma \partial_z \left( B_x - \tau \partial_t B_x \right) + \eta \partial_z^2 B_x \]
Time-delay dynamo for Roberts II and III flows

\[ \partial_t B_x = -\gamma \partial_z (B_x - \tau \partial_t B_x) + \eta \partial^2_z B_x \]

\[ \text{Re} \lambda = \frac{\tau (\gamma k)^2 - \eta k^2}{[1 + (\tau \gamma k)^2]} \]

Growth when
\[ \tau > \frac{\eta}{\gamma^2} \]
\[ \tau \nu_{\text{rms}} k_f > \frac{1}{3} \]

Rheinhardt et al. (2014)
Conclusions

• Small scale deep convection
• Deep convective flux: Deardorff
• Thus marginally stable (not unstable)
• Such flows yield weaker turb diffusion
• Favor spot formation by NEMPI
• Dynamo effect from time delay