Introduction

Goals: In this lab we explore effects of atmospheric drag on motion of satellites that are in low enough orbits to be affected by the Earth’s atmosphere. Our goals are to understand:

1. Orbital Mechanics including simple drag.
   a. Applications to satellite operations

Methods: In previous labs we have dealt with the output of numerical simulations. Here we try something different: Analytical and Empirical Models. We will use “pencil and paper” calculations as well as visualization of the MSIS Atmosphere model invoked by IDL

Motivation: Satellite drag has a direct affect on a space operators’ ability to:
   • Track and identify active payloads and debris
   • Avoid collisions and make re-entry prediction
   • Characterize the atmosphere’s density (and temperature) profiles

Activities Supporting the Goals

• Develop basic physics to describe satellite motion
  Activity: Orbit Speed, Drag, Work, Height Calculation Worksheet
• Investigate behaviors of satellites (in near circular orbit) under the influence of drag by plotting diagrams of Work/energy, Drag force, Velocity, Altitude vs time
  Activity: Simple Orbit Decay with MSIS Atmosphere
• Compare and contrast ideal and model atmospheres
  Activity: MSIS Altitude Profiles
  Activity: MSIS Global Mapping
• Explore effects of time-varying atmosphere heating
  Activity: STARSHINE comparison

Pre-Lab Thinking and Concepts

Draw a time height profile for a satellite in circular orbit in the low earth atmosphere
(How does the radius of the orbit change with time?)
Draw a time speed profile for a satellite in circular orbit in the low earth atmosphere
**Satellite Motion Review**

Simplified Concept Map:

- Boundary Conditions and Governing Eqns
- Work done by Drag Force
- Reduced Mechanical Energy
- New Conditions but Same Physics
- More Work done by Drag Force
- Satellite De-Orbits

**Using Your Brain for Satellite Energy and Drag Application**

Activity: Orbit Speed, Drag, Work, Height Calculation Worksheet

- Develop and use basic physics to describe satellite motion
  *Pencil, paper, calculator and Lab Worksheet*

![Image](https://example.com/image1.png)

The Moon rising above the limb of the Earth. Seen through scattered sunlight, the atmosphere fades dramatically to the black background of space. (NASA Photo)

The approach taken by the IDL program is iterative. You will do one baseline calculation and one iteration.

**Values for worksheet calculations:**

*Orbital Drag Lab: Values for Space Shuttle Orbit*

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m ), Mass of Shuttle (kg)</td>
<td>91974</td>
</tr>
<tr>
<td>( C_D ), Drag Coefficient</td>
<td>2</td>
</tr>
<tr>
<td>( A ), Cross Sectional Area of Shuttle (m²)</td>
<td>362</td>
</tr>
<tr>
<td>( h_i ), Initial Altitude (km)</td>
<td>350</td>
</tr>
<tr>
<td>( G ), Gravitational Constant (Nm²/kg²)</td>
<td>6.67E-11</td>
</tr>
<tr>
<td>( M ), Mass of Earth (kg)</td>
<td>5.97E+24</td>
</tr>
<tr>
<td>( R_e ), Radius of Earth (m)</td>
<td>6.37E+06</td>
</tr>
</tbody>
</table>

MSIS standard atmospheric density at 350 km = 4.25x10⁻¹¹ kg/m³

**DO THIS:** Calculate all values in top row of the worksheet, then calculate \( E \), altitude, and speed for next orbit (second row). See next page for help

Check your answers against answer sheet (No peeking until you’re done)
Basic Newtonian Mechanics

Kinetic and Potential Energy

• Satellites in orbit experience a centripetal acceleration
  \[ \ddot{r} = m\ddot{a} = -\frac{mv^2}{r}, \quad \dot{r} = \frac{GMm}{r^2} \]

• Solve for speed
  \[ v = \sqrt{\frac{GM}{r}} \]

• Associated kinetic energy
  \[ \frac{1}{2}mv^2 = \frac{mGM}{2r} \]

• Potential Energy
  \[ \Delta U_{AB} = \int_{A}^{B} F \cdot d\vec{r} = -\frac{GMm}{r} \]

Significance of “-” sign?

Basic Newtonian Mechanics

Total Energy and Work

• Total Mechanical Energy
  \[ E = \frac{1}{2}mv^2 + -\frac{mGM}{r} = -\frac{GMm}{2r} \]

• Solve for Altitude
  \[ r = -\frac{GMm}{2E} = R_E + h \]

• Note: Total Mechanical Energy is constant unless non-conservative forces acts

• Drag Force
  \[ |F_D| = \frac{1}{2} \rho AC_D v^2 \]

• Work by Drag Force
  \[ W = \int F_D \cdot \vec{r} = \frac{1}{2} \rho AC_D v^2 l \]
Review of the Theory of the Law of Atmospheres

First we need to know something about the atmosphere these satellites are moving through. The simple law of atmospheres states that, close to the earth's surface, the atmospheric density decreases exponentially with elevation. The equation we use is derived from a hydrostatic assumption:

\[ \rho(y) = \rho_0 \exp(-mg \cdot y/kT) \]

where
\( \rho \) is the mass density (kg/m\(^3\)),
\( \rho_0 \) is the mass density at sea level, (kg/m\(^3\))
\( m \) is the mass of a gas molecule, (kg)
y is the elevation above sea level (m)
k is the Boltzmann gas constant (J/K)
\( T \) is the temperature in kelvins.

This expression assumes that the acceleration due to gravity \( g \), the temperature \( T \), and the mean gas molecule mass, \( m \), remain constant.

MSIS Model Intro/Review

The MSIS empirical model provides

- Global thermospheric temperature and density of N\(_2\), O, O\(_2\), He, Ar, and H

Based on \textit{in-situ} data from 7 satellites, numerous rocket probes, radar measurements and \textit{theory}.

Domain: 0-1000 km, includes local time, seasonal, and solar cycle-effects

Important Model Parameters:
- F10.7 radio wavelength proxy for EUV heating associated with photoionization
- Ap daily, linear, planetary proxy for geomagnetic activity
- Global Mid-latitude--geomagnetic energy typically provides 20%-- more during storms

Using the MSIS Atmosphere Application

Logon to your computer, open a terminal window and go to the lab 8 directory
[cd swss-labs/lab8]. Start IDL by typing “idl”.

Note commands are invoked by \texttt{IDL> cmd}

- Compare and contrast ideal and model atmospheres

\textbf{Activity: MSIS Altitude Profiles} \hspace{1em} (See Figure below)

\texttt{IDL> atmosphere, /TEMP}

Successive clicks on the graph will reveal different models

Q: Which model shows the highest density at 800 km?

Q: Which model shows the highest temperature at 800 km?

Suppose, rather than height profiles, you would like to see a global view of atmospheric variations and further you’d like to be able to specify different background conditions
Activity: MSIS Global Mapping

IDL> msisatmosphere

Use a new program here to get representations of atmospheres with
differing locations
differing seasons
variable solar drivers (F10.7)
variable geomagnetic drivers (Ap)

DO THIS: Now is a good time to explore the msisatmosphere panel and widgets and answer the guided questions on next two pages

Guide Questions for MSIS Atmosphere Activities:

COMPARE TEMPERATURE and MASS DENSITY PROFILES

1. Start with the lowest F10.7 and Ap and make no changes to location or time.
   a. Choose altitude plot from the pull down menu
   b. Choose the variable: mass density (md)

   What is the temperature at 800 km? ________________
   What do you notice about the rate of temperature change above 250 km (lapse rate)? ______________________
   What is the value of mass density at 800 km (and units)? ________________

2. Now change F10.7 to 250.

   What is the temperature at 800 km? ________________
   What is the value of mass density at 800 km? ________________
COMPARE NUMBER DENSITY PROFILE

3. Now change F10.7 to 67.
   a. Choose altitude plot
   b. Choose the variable: O (atomic Oxygen)

   What is the value of number density at 800 km (and units)? _____________

   a. Choose altitude plot
   b. Choose the variable: O (atomic Oxygen)

   What is the value of number density at 800 km (and units)? _____________

   Why do you think this variation occurs?
   _________________________________________________________________

INVESTIGATE GLOBAL MAPPING AT THE SURFACE

5. Now change F10.7 to 67.
   a. Choose PLOT TYPE = map
   b. Choose VARIABLE TYPE=Temp
   c. Choose time = 00 hr
   d. Choose MAP TYPE = Ortho Mapping
   e. In the bottom-most double blocks enter 40 for LAT and -70 for LON (Boston) and click on RANGE=AUTO
   f. Choose altitude = 0 km

   Is this the temperature structure you expect? ______________

   Does anything change at the surface if you maximize F10.7? ______

   Is this what you expect? ________________________________

INVESTIGATE GLOBAL MAPPING NEAR THE TROPOPAUSE

6. Now minimize F10.7 again
   a. Choose altitude = 12 km

   How has the temperature structure changed with respect to 0 km?
   _____________________________________________________________
INVESTIGATE SEASONAL VARIATION NEAR THE TROPOPAUSE

b. Change the date to 1990001
What time of year is it in Boston? How can you tell from the temperature profile?

INVESTIGATE TEMPERATURE HEIGHT VARIATIONS

c. Now click to the right in the altitude box….the altitude rises in increments of 100 km. (You may need customize the contour spacing in the double boxes above LAT and LON)

How has the temperature structure changed? ______________

d. Now increment the altitude several times

At what altitude does the atmospheric temperature become constant? ______________

e. For that altitude maximize F10.7

What happens to the temperature? ______________

Are there longitudinal variations in the temperature? ______________

f. What time of year is it?

g. What is the local time in Boston?

Change the Map Type from “ortho” to “cyl” or “mollweide” to help answers these questions.

h. Minimize F10.7 and maximize Ap

What happens to the temperature? ______________

Are there latitudinal variations in the temperature? ______________

IDL>EXIT

Take a break if you need one

Return from break
Using the orbitdecay Application
Explore effects of time-varying atmosphere heating using STARSHINE satellite data

The STARSHINE satellites, of which there were three, were launched with the intention of gathering information about satellite drag. These flew at various times in the late 1990’s and early 2000’s. They were (nearly) spherical satellites covered with mirrors.

Investigate behaviors of satellites (in near circular orbit) under the influence of drag by plotting diagrams of Work/energy, Drag force, Velocity, Altitude vs time

Activity: Simple Orbit Decay with MSIS Atmosphere
IDL> orbitdecay

For the Starshine 2 satellite, compare effects of different activity levels over lifetime of mission

- Use the following entries for F10.7 and Ap. As you make the entries, think about the level of activity that each combination represents. Note time must go to 80,000 hrs (It’s a feature!).
- Hit the PLOT DATA button.
- Choose and view all the plot types on the button below the “Plot Data” button.

<table>
<thead>
<tr>
<th>Case</th>
<th>Solar 1</th>
<th>Auroral 1</th>
<th>Solar 2</th>
<th>Auroral 2</th>
<th>Solar 3</th>
<th>Auroral 3</th>
<th>Solar 4</th>
<th>Auroral 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F10.7 #1</td>
<td>Ap#1</td>
<td>F10.7 #2</td>
<td>Ap#2</td>
<td>F10.7 #3</td>
<td>Ap#3</td>
<td>F10.7 #4</td>
<td>Ap#4</td>
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<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
</tbody>
</table>

In which atmosphere does a satellite maintain the longest orbital lifetime?
In which atmosphere does the satellite de-orbit most rapidly?
Describe how the velocity and force curves change with time.

**DO THIS:** You can enter as many as 5 epochs of activity combinations, try 0, 10,000, 20000….80,000 or any combination that ends at 80,000 hr. Notice how the character of the smooth curves changes.

Select the “Overplot Starshine 2 Trajectory” button and see how well your altitude profile fits that of the actual satellite data.

**DO THIS:** Can you change the F10.7 and Ap entries in orbitdecay so that you match the observed satellite orbital decay for any one satellite?

For STARSHINE satellites (Varying temperature or activity level over the mission lifetime)

<table>
<thead>
<tr>
<th>Time</th>
<th>Starshine #1</th>
<th>Starshine #2</th>
<th>Starshine #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>TBD1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>30,000</td>
<td>TBD1</td>
<td>3</td>
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<tr>
<td>3</td>
<td>30,500</td>
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<td>3</td>
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<tr>
<td>4</td>
<td>80,000</td>
<td>TBD1</td>
<td>TBD5</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on the values you need to enter for F10.7 and AP, do you think this satellite was orbiting during solar max or solar min? Why?
Post-Lab Thinking and Concepts

Draw a time height profile for a satellite in circular orbit in the low earth atmosphere
Draw a time speed profile for a satellite in circular orbit in the low earth atmosphere
Draw a time-force profile for a satellite in circular orbit in the low earth atmosphere
Draw a time-energy profile for a satellite in circular orbit in the low earth atmosphere

How does the speed change affect a space operator’s ability to track the satellite?

Will a de-orbiting satellite be ahead of or behind schedule for passing over a fixed point on the earth?

Note: We’ve assumed the density is constant over the course of the orbit….in general this is not the case. In advanced simulations we would relax this assumption and also deal with non-circular orbits.