Joy’s Law: A Space-Age Update
Aimee Norton, Stanford University, SDO/HMI

- Bipolar magnetic regions exhibit a tilt with respect to E-W direction
- Follower (f) is farther from Equator than preceding (p) spot
- Tilt increases at higher latitudes

HMI data, NOAA 11428, 2 Mar 2012
−17° Latitude, 550 × 375 pixels, sharp_cea_720s data
1st Report: Sunspot Group Tilt Angle Depends on Latitude

- Hale, Ellerman, Nicholson & Joy (1919) (tilt itself reported prior)
- 2633 sunspot groups from $I_c$ drawings, Carrington & Spörer
- Data from 1856-93, Cycles 10, 11, 12 and 13 partial
- No scatter reported, deemed ‘Joy’s Law’ by Zirin 1988

**Definition of tilt:**
$$\tan(\alpha) = \frac{\Delta \theta}{\Delta l \cos(\theta)}$$

\begin{align*}
\alpha &= \text{tilt, } \theta = \text{latitude} \\
\Delta \theta &= \Delta \text{lat for p and f spot} \\
\Delta l &= \Delta \text{lon for p and f spot}
\end{align*}

**Forms of Joy’s Law**
$$\alpha = m_A \theta$$
$$\alpha = m_A \sin(\theta) + c$$
Whirling storms in the earth’s atmosphere, whether cyclones or tornadoes, follow a well-known law which is said to have no exceptions: the direction of whirl in the Northern Hemisphere is left-handed or counterclockwise, while in the Southern Hemisphere it is right-handed or clockwise. The theory of terrestrial cyclones is still very obscure, but the direction of whirl is evidently determined by the increase in linear velocity of the air from pole to equator, due to the earth’s rotation. The question naturally arises whether storms in the solar atmosphere are also whirlwinds, and, if so, what law governs their direction of whirl in the Northern and Southern hemispheres.
Sunspots & groups display much less rotation than terrestrial cyclones. AR 11158 from Feb 2011 as observed by SDO/AIA and HMI (Dan Brown and)

But these are anomalies. Most spot groups have a tilt, not a visible rotation.
Joy’s Law: The Paradigm

Rising flux tube + Coriolis force acting on flows produces tilt + Turbulent convection produces scatter in tilt

Modification to cartoon above: The retrograde flow in the tube at depth sets up an asymmetry that causes leading and following legs to be different; therefore, follower leg more radial.

D’Silva & Choudhuri 1993
Caligari et al 1995
Also Fan & Fisher 1996
Joy’s Law: The Paradigm and the Possibilities

An inherent geometric property of interior field, like a rope always twisted in one direction, ie “pitch angle”

A mean kinetic helicity imparted during rise through convection zone by large-scale, subsurface flow gradients imparting a net tilt.

What determines tilt in thin flux tube simulations?

Rise Time (1-8 months) – and associated flows in tube. Rise time is a function of flux. Flux tube radius, Drag force, External flow field (upflows, etc)
Wang & Sheeley (1989) studied tilt angles from 2700 bipolar regions ($\Phi_{B\text{leader}} > 3 \times 10^{20} \, \text{Mx}$) in NSO/KP magnetograms, 1976-86 (Cycle 21). Binned (5°) average tilt angle values (left) and RMS (right) for strong (thin solid), medium (dash) and weak (dot) bipolar magnetic regions (not just sunspots). Thick line is flux-weighted tilts. Note weak regions have more scatter, probably due to tubes with reduced flux being more strongly influenced by convective flows.

So many good studies already done –


**More good studies**

*Howard, 1996* plage tilts 2x sunspots. Tilts tend towards average tilt, not 0.

*Fisher, Fan & Howard (1995)* find higher tilts for spots with greater separation, higher scatter for low flux regions.


*Kosovichev & Stenflo (2008); SK (2012)* MDI data, Tilt settles to Joy’s law value, not 0. No dependence of Joy’s Law slope on magnetic flux.

*D’Silva & Choudhuri (1993); Schussler et al (1994)- numerical simulations*, find $B_0$ of 60 - 160 kG is needed to match observed tilts.

*Fan, Fisher & McClymont (1994)* predict $\alpha \approx \sin \theta \Phi^{1/4} B_0^{-5/4}$

$\alpha = \text{tilt}, B_0 = \text{toroidal field strength}, \Phi = \text{flux in tube}$
The only space-age data bases from MDI and HMI are being compiled and maintained by Debrecen, others do not exist or are not public.

*Public Tilt Angle Database*

<table>
<thead>
<tr>
<th>Data</th>
<th>Dates</th>
<th>Cadence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mount Wilson (MW)</td>
<td>1917–1985</td>
<td>daily</td>
</tr>
<tr>
<td>Kodaikanal (KK)</td>
<td>1906–1987</td>
<td>daily</td>
</tr>
<tr>
<td>Debrecen Photoheliographic Data (DPD)</td>
<td>1974–2014</td>
<td>daily</td>
</tr>
<tr>
<td>SOHO/MDI-Debrecen Data (SDD)</td>
<td>1996–2010</td>
<td>hourly</td>
</tr>
<tr>
<td>SDO/HMI-Debrecen Sunspot Data (HMIDD)</td>
<td>2010–2013</td>
<td>hourly</td>
</tr>
</tbody>
</table>

*STARA catalog (Watson) is public but does not contain tilt angle.*

*Kosovichev and Stenflo analysis - MDI tilt data not public.*

*Li and Ulrich analysis - MWO magnetogram tilt data not public.*

*No Kitt Peak tilt angle archives (that I know of).*
The resulting slope of Joy’s Law from $I_c$ is lower (0.28) than from $B_{\text{los}}$ (0.46), in part from improper identification of umbral polarity but inclusion of plage changes the value. “Plage are more highly tilted as their rise time is longer.”
Anti-Hale (AH) spots – traditionally, misidentify them in $I_{c}$

8.4% of regions are AH (McClintock, Norton & Li, 2014)
more at equator than elsewhere due to magnetic equator not aligning with heliographic
6.9% of groups were AH in Weber et al 2013 simulations =>
tubes emerging in opposite hemisphere due to interaction with convective flows
$\approx$4% reported by Wang & Sheeley 1989 and Stenflo and Kosovichev 2012

Use of $I_{c}$ to assign group & tilt misses AH spots

Adding Tilt to Butterfly Diagram

Without Anti-Hale - DPD

With Anti-Hale – Mt Wilson
Some interesting questions to consider …

• What does the tilt vs time for an individual active region tell us?

• What do we expect to see at the Equator?

• Are anti-Hale / anti-Joy spots inherently different or are they just the tails of a distribution?

• Does the tilt depend on the amount of flux?

• Do active regions originate from a smooth, slab-like geometry, or is the flux already in a flux tube form at depth?
Joy’s Law from several authors

Note: 2 lines go through the origin, others have y-intercept

Figure from McClintock & Norton 2014

See poster here by V.Senthamish Pavai

\[
\sin \gamma = 0.48 \sin \theta + 0.03
\]
\[
\sin \gamma = 0.5 \sin \theta
\]
\[
\gamma = 0.2 \theta + 2.0
\]
\[
\gamma = (0.26 \pm 0.05) \theta
\]

Wang & Sheeley, 1991 Kitt Peak Magnetograms 2710 regions Cycle 21
Leighton, 1969 Zurich Observatory data Brunner, 1930 Cycle 16
Norton & Gilman, 2005, 650 regions, MDI Intensity data, Cycle 23
Dasi-Espuig et al., 2010, Mt Wilson Observatory, Cycle 15-21

\[
\gamma_N = 0.21 \theta + 1.6
\]
\[
\gamma_S = 0.15 \theta + 1.3
\]

N & S hemispheres separately determined, Mt Wilson, Cycle 15-21
Mean Tilts and Joy’s Law Slope Varies Significantly with Cycle and Hemisphere – Absent in N17, S19

Tilt separated by hemisphere (North=red, South=green) (McClintock & Norton, 2013)

Joy’s Law too noisy (or absent) for individual cycles and hemispheres. Mean tilts show significant variation cycle to cycle and between hemispheres.
Tilts Inversely Correlated with Cycle Strength

Dasi-Espuig et al 2013 (corrigendum)

Tilt Angle inversely correlated with cycle strength

Since a higher tilt angle can produce a stronger polar cap field strength, this finding may indicate a self-correcting system.
Hemispheric Tilts Correlated with Cycle Strength

McClintock & Norton 2013, Cycle 19 outlier
Weber, Miesch and Fan (2012); Weber and Fan (2015) – Thin flux tube simulations that include convection & radiative diffusion. Recover slope of Joy’s law as a function of toroidal field strength for a $10^{22}$ Mx flux in rising tube…

<table>
<thead>
<tr>
<th>$B$ [kG]</th>
<th>$m_{A-AB}$</th>
<th>$m_{A-RD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.25 ± 0.02</td>
<td>0.32 ± 0.02</td>
</tr>
<tr>
<td>80</td>
<td>0.29 ± 0.02</td>
<td>0.40 ± 0.02</td>
</tr>
<tr>
<td>60</td>
<td>0.35 ± 0.02</td>
<td>0.44 ± 0.04</td>
</tr>
<tr>
<td>50</td>
<td>0.38 ± 0.03</td>
<td>0.42 ± 0.04</td>
</tr>
<tr>
<td>40</td>
<td>0.40 ± 0.04</td>
<td>0.27 ± 0.05</td>
</tr>
<tr>
<td>30</td>
<td>0.31 ± 0.06</td>
<td>0.32 ± 0.05</td>
</tr>
<tr>
<td>15</td>
<td>0.25 ± 0.07</td>
<td>0.31 ± 0.07</td>
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Flush tubes with longer rise time (ie prolonged time in convection zone), encounter multiple convection cells and are influenced more by the mean kinetic helicity rather than stochastic fluctuations and result in higher slope.

Also produce RMS deviation so can compare with observed scatter.
Can negative tilts, large scatter and anti-Hale spots be explained in a thin flux tube, deep seated paradigm? Yes

Weber et al (2012) find that for weak toroidal field strengths (15-30 kG) and low fluxes (\(10^{20}\) Mx), the loop apex hosts a converging flow in upper convection zone and produced negative tilts.

Anti-Hale spots occurred (6.9% of total spots) due to flux tubes crossing hemispheres due to large-scale convective flows.

But there are still concerns

Can a flux tube stay coherent during its rise? Doesn’t a small amount of flux pinched off at \(R=0.7\) become a GIANT, expanded, blob at \(R=1\)?

Wouldn’t the footpoint separation at the photosphere be greater than 25-70 Mm for most thin flux tubes rising from the base of the CZ?
Near-Surface Dynamo / NEMPI
Explanation of Joy’s Law
Smallest bipoles show negative tilts Ilarionov, Tlatov, Sokoloff 2015; Tlatov et al (2013)

Bipoles smaller than 300 MSH (or Flux < 10^{21} Mx) display different behavior. For both hemispheres, positive angle means leading polarity closer to equator.
An important question to address is the effect of evolutionary processes on the measured tilt angles. Brunner (1930) noted that the axial inclinations of emerging sunspot groups decrease with time due to the westward proper motions of their leading spots. Not clear if TFT simulations reproduce observed footpoints separations.
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Relationship Between Supergranules & Bipolar Active Regions

Sunspot flux emerges in a region smaller than supergranule diameter.

Footpoints of bipolar magnetic regions are on average 25 Mm when first observed in Ic and reach max distance of 75 Mm – same size as SG.

Footpoint separation speed (180-250 m/s) similar to horizontal divergent flow of SG horizontal velocities on surface. Vz very different, though.

Divergent flows both subject to Coriolis force.
Conclusions -96 Years Later, Still Interesting

1) Revise Joy’s law to have lower slope (~0.16-22). Don’t force to 0 at equator.

2) Tilts and slopes vary with cycle and hemisphere. (Time periods when Joy’s law is absent are most interesting.)

3) Whole-Sun, Cycle 17-21 data suggests stronger cycles produce smaller tilts.

4) Use of $B_{los}$ (not $I_c$) to determine tilts is changing our understanding. AH spots more numerous than previously thought (8%). Careful compilation of tilt angle data is necessary.

5) Numerical simulations of thin flux tubes rising through CZ with convection and rad cooling have gained traction with reasonable reproduction of tilts.

6) Need more studies that include small bipoles, and a better understanding of how tilt changes during lifetime of AR.
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   Stop using Ic data and instead, analyze B only to discern tilt.

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But there are still concerns with thin flux tube simulations?

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