Spatially coupled inversion of spectro-polarimetric data

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Inverting solar data
Response Functions

How to adjust parameterized atmosphere?

\[ J_{x,y} \cdot \Delta \alpha_{x,y} = \Delta S_{x,y} \]

\[ J_{i,\lambda}^{i,\lambda} = RF_{x,y}^{i,\lambda} = \frac{\partial S_{x,y}(\lambda)}{\partial \alpha_{i,x,y}} \]
The coupled reality
Telescope diffraction

For given fixed psf $\varphi(x, y)$, the observed intensity $d(x, y)$ of a point source at $(x_0, y_0)$ is given by:

$$d(x, y) = \varphi(x - x_0, y - y_0)S(x_0, y_0)$$

Response:

$$\frac{\partial d(x, y)}{\partial \alpha_i} = \varphi(x - x_0, y - y_0) \frac{\partial S(x_0, y_0)}{\partial \alpha_i}$$

Response at $(x_0, y_0)$ spreads to other pixels!
Coupled response matrix

For a uniform PSF, the pseudo-inverse $J^TJ$ has the form

$$J^TJ = \begin{pmatrix} Y_{0,0}J_{1,1}^TJ_{1,1} & \cdots & Y_{1-n,1-m}J_{1,1}^TJ_{n,m} \\ Y_{0,1}J_{1,2}^TJ_{1,1} & \cdots & Y_{1-n,2-m}J_{1,2}^TJ_{n,m} \\ \vdots & \ddots & \vdots \\ Y_{n-1,m-2}J_{n,m-1}^TJ_{1,1} & \cdots & Y_{0,-1}J_{n,m-1}^TJ_{n,m} \\ Y_{n-1,m-1}J_{n,m}^TJ_{1,1} & \cdots & Y_{0,0}J_{n,m}^TJ_{n,m} \end{pmatrix}$$

where $Y$ is the autocorrelation function of the PSF

$$\sum \sum \varphi_{k,l} \varphi_{k-i,l-j} = (\varphi \ast \varphi)_{i,j} = Y_{i,j},$$

This matrix is sparse!
Coupled Inversion

How to solve the pseudo problem \((\mathbf{J}^T \mathbf{J}) \cdot \Delta \alpha = \mathbf{J}^T \cdot \Delta \mathbf{S} = \beta\) ?

- 3 height nodes \(\times (T, B, \gamma, \chi, v_{los}, v_{mic})\)
- ”Typical” FOV: 300x300 pixels
- ”Typical” application: 3 \(\cdot\) 10^6 fit variables
- \(\rightarrow\) Direct matrix inversion impossible (\(N \gg 1000\))!

Then what?

- Forward matrix is sparse
- Improve ”guess” value \(\Delta \alpha_0\) iteratively
  - Calculate \(\beta_0 = (\mathbf{J}^T \mathbf{J}) \cdot \Delta \alpha_0\)
  - Approximate: \(A^\dagger = \text{approx} \{\mathbf{J}^T \mathbf{J}\}\)
  - Calculate \(\delta(\Delta \alpha) = (A^\dagger)^{-1}(\beta - \beta_0)\)
  - Apply correction \(\Delta \alpha_1 = \Delta \alpha_0 + \delta(\Delta \alpha)\)
  - repeat
Application

- Constant PSF: Hinode SP scans
- FeI 6301.5\AA{} + 6302.5\AA{}
- 3 height nodes ($\log \tau = -2.0, -0.8, 0.0$)
- one atmosphere per pixel
- PSF: 10”x10” ($\sim 99\%$ of the light accounted for)

- No stray-light (off-limb estimate: $\sim 1\%$)
Stokes I before (left) and After (right) inversion
Stokes V before (left) and After (right) inversion
Conclusions

▶ Spatial smearing matters (a lot)
▶ Complex profiles result of spatial convolution
▶ Fits stray-light insensitive
▶ Coupled inversion gives "optimal" deconvolution (given the model)

Important to remember:
▶ Coupled inversion gives coupled solution
▶ Errors also coupled!
▶ Error bars not very meaningful